SUPPLY NETWORK FORMATION AND FRAGILITY

Matthew Elliott
Cambridge

Benjamin Golub
Northwestern

Matthew V. Leduc
PSE

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The robustness of a complex supply network

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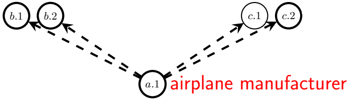

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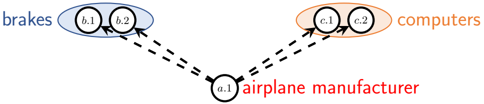

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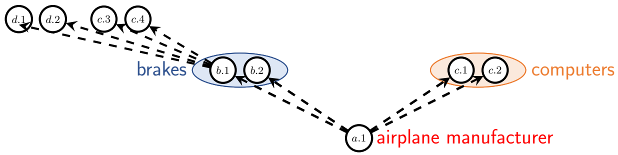

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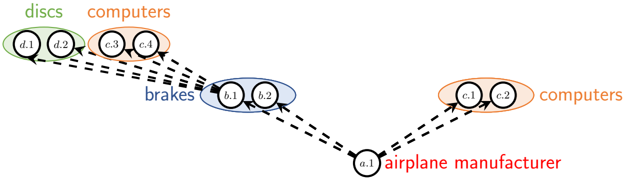

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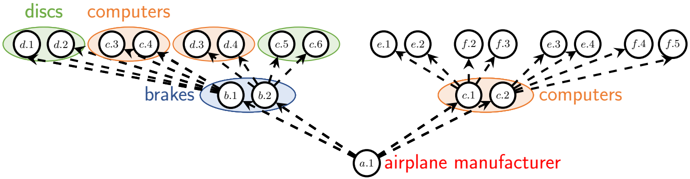

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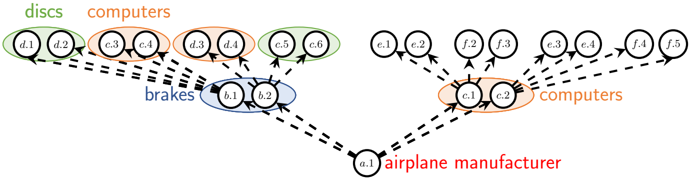

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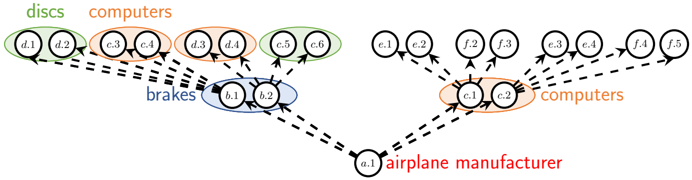

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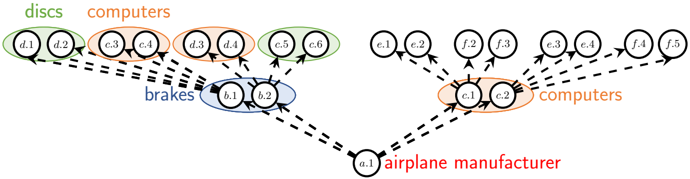

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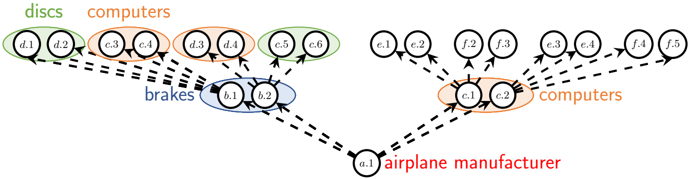

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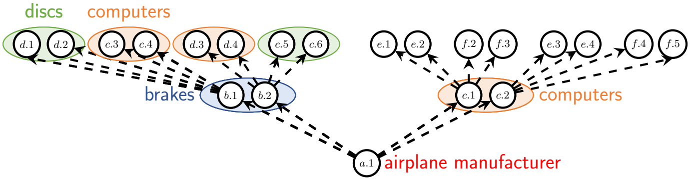

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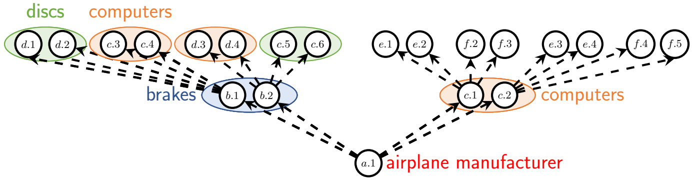

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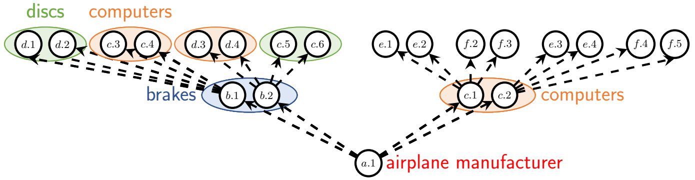

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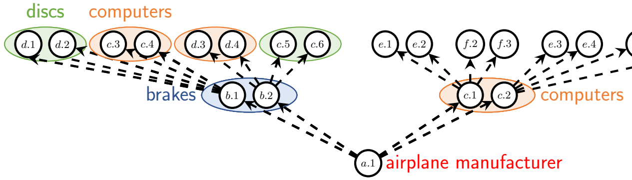

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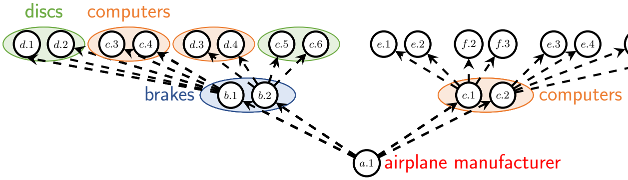

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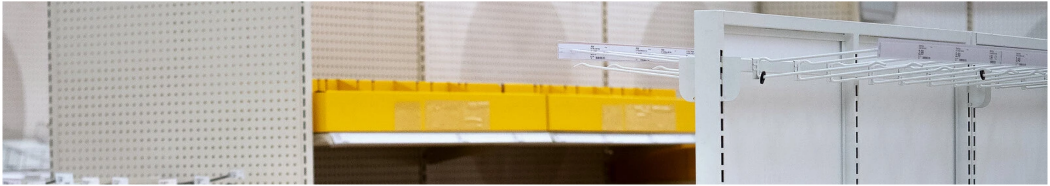
- Inefficient.

How the World Ran Out of Everything

Global shortages of many goods reflect the disruption of the pandemic combined with decades of companies limiting their inventories.

The New York Times



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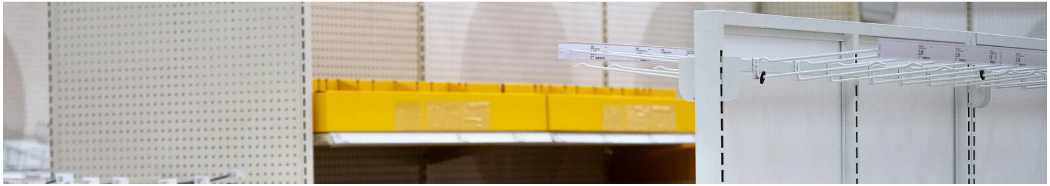

– *In normal times, the company is behind in filling perhaps 1 percent of its customers' orders. On a recent morning, it could not complete a tenth of its orders because it was waiting for supplies to arrive. The company could not secure enough of a specialized resin that it sells to manufacturers that make construction materials. The American supplier of the resin was itself lacking one element that it purchases from a petrochemical plant in China. One of Mr. Romano's regular customers, a paint manufacturer, was holding off on ordering chemicals because it could not locate enough of the metal cans it uses to ship its finished product. "It all cascades," Mr. Romano said. "It's just a mess."*

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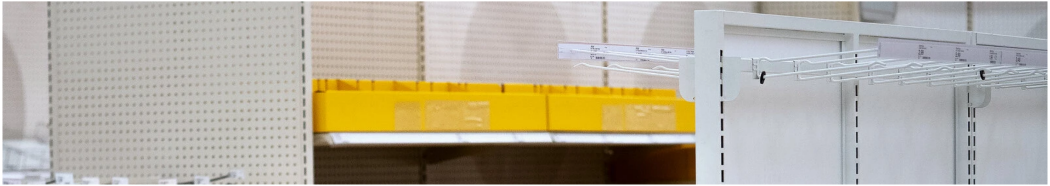

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- *Simply expanding warehouses may not provide the fix... Product lines are increasingly customized. The ability to predict what inventory you should keep is harder and harder.*

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Model agents (firms) endogenously investing in their links. Interesting mechanics and welfare issues.

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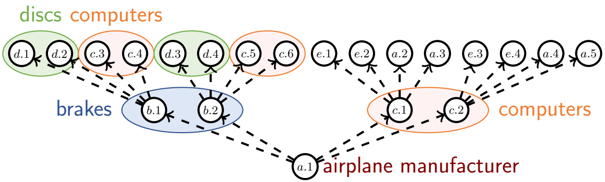

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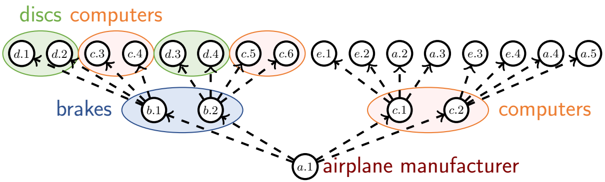

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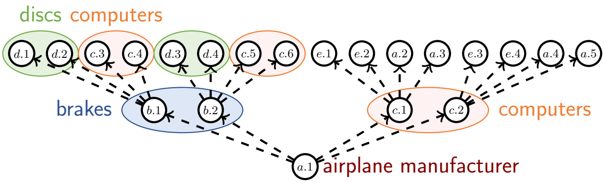

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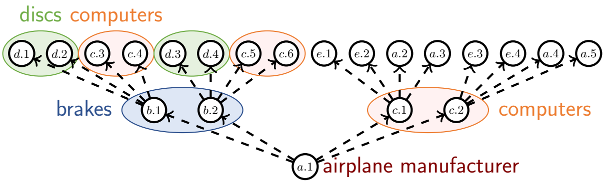

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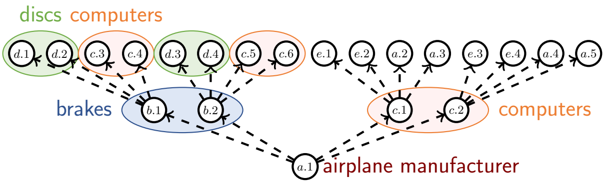
$\widetilde{\rho}(x, d)$ – probability that firm $a.1$ can produce; **"reliability"**



The supply network upstream of one firm

A SINGLE SUPPLY TREE

Small firms have supply relationships to other small firms. Typical firm needs to procure several inputs to produce.

- Has **multiple** potential sources of each input. Each link works with probability x, independently.
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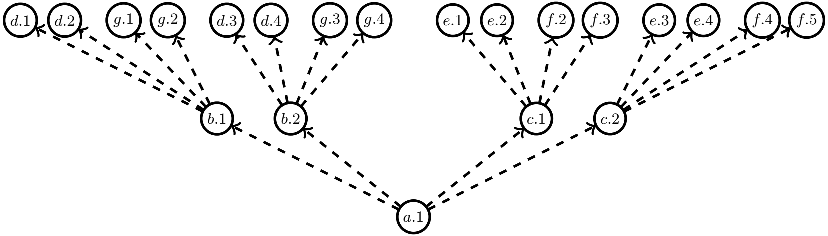

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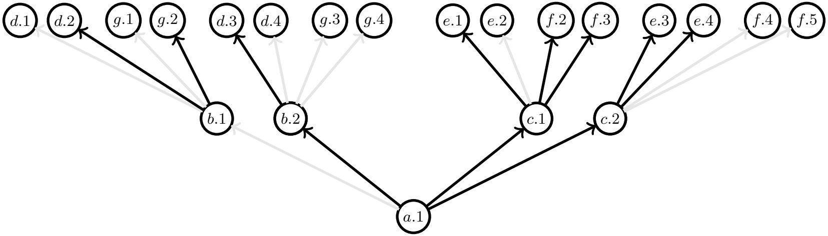

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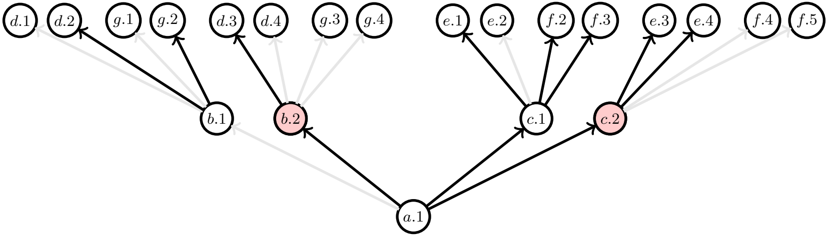

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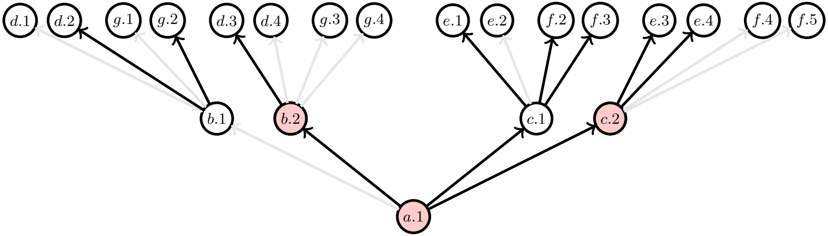

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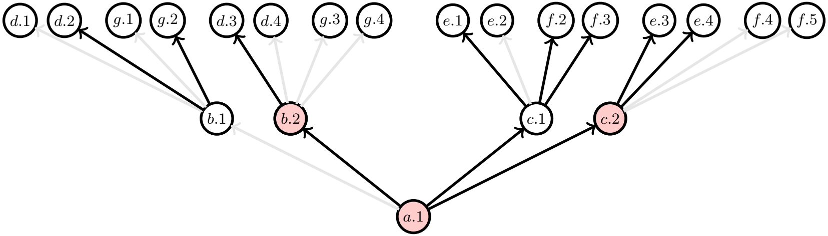

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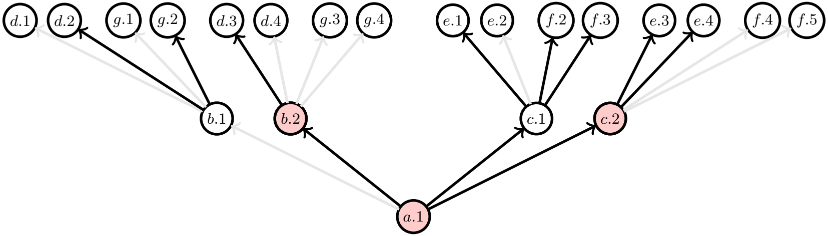

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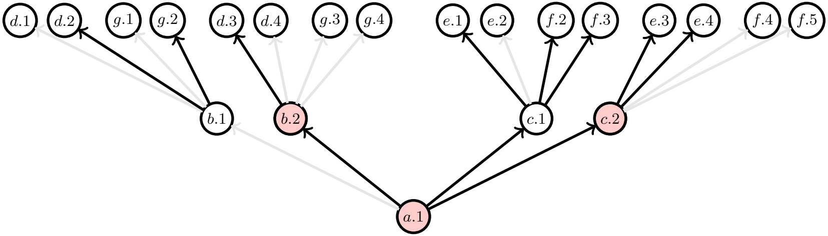

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Realized supply network $\mathcal{G}$: links in $\mathcal{G}'$ are **operational** independently with probability x.

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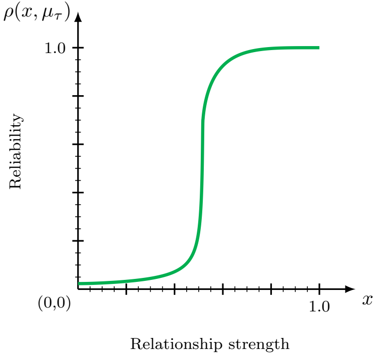

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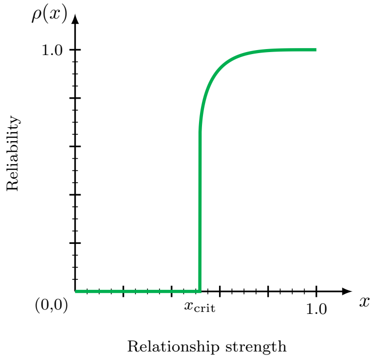

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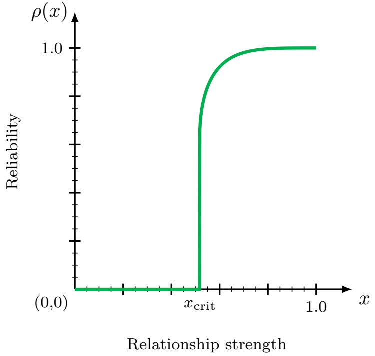

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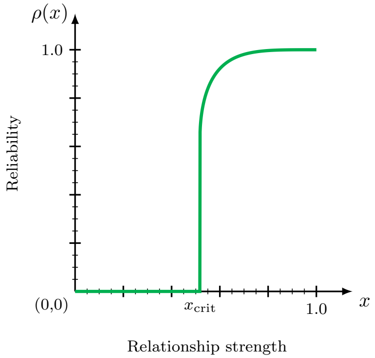

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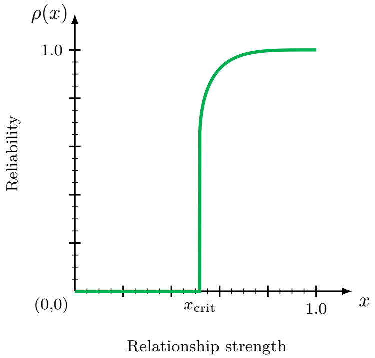

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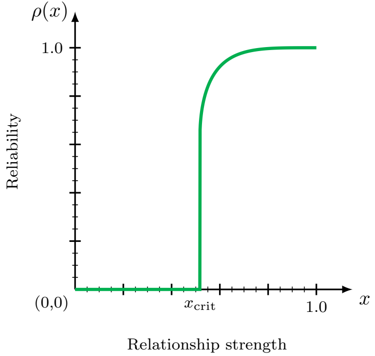

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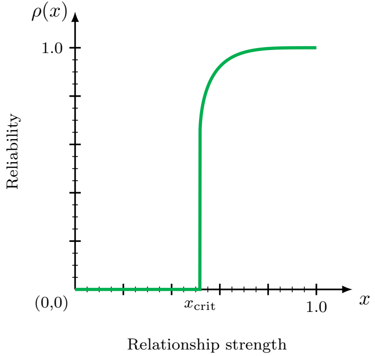

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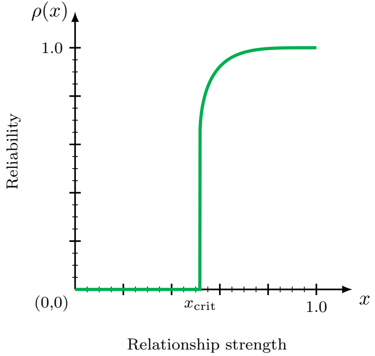

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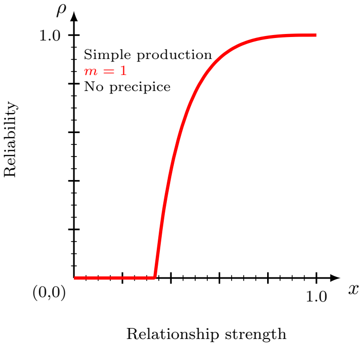

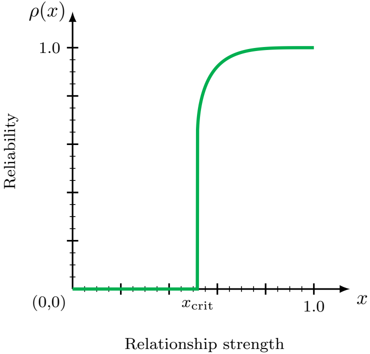

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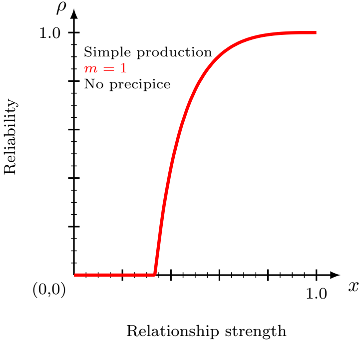

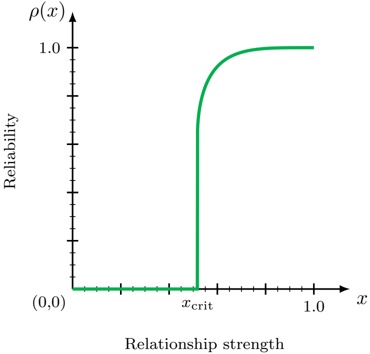

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This does not happen for simple production, $m = 1$.

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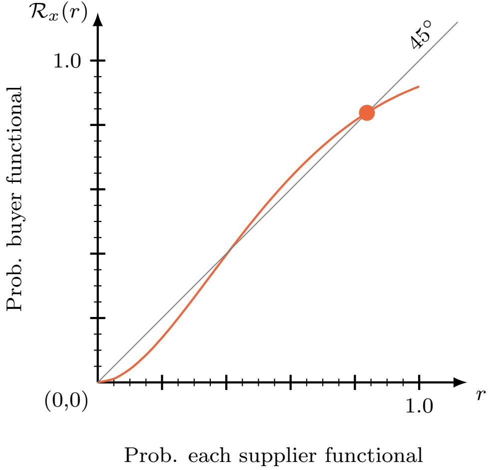
Understanding the precipice

Part 1: How reliability depends on supplier reliability

- Suppose each of your suppliers is functional with probability r independently.



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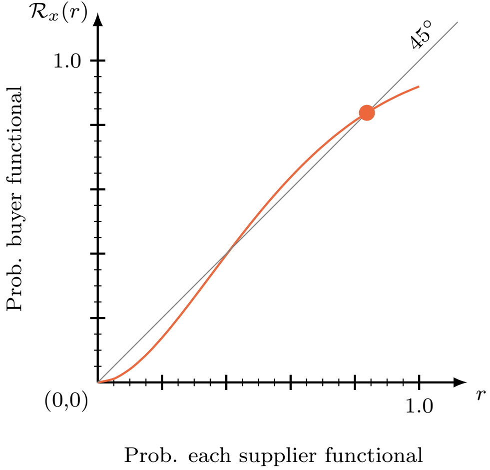
- Let $\mathcal{R}_x(r)$ be the probability that you are functional.

$$\mathcal{R}_x(r) = \left[\quad \left(\quad\quad \right) \quad \right]$$



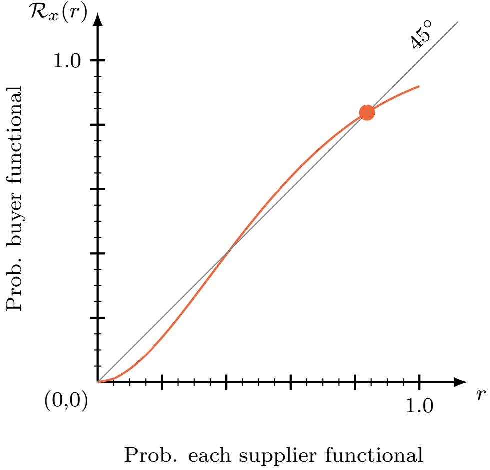
Understanding the precipice

Part 1: How reliability depends on supplier reliability

- Suppose each of your suppliers is functional with probability r independently.
- Let $\mathcal{R}_x(r)$ be the probability that you are functional.

$$\mathcal{R}_x(r) = \left[\qquad \left(\qquad xr \right) \; \right]$$

Prob. given supplier is available



Understanding the precipice

Part 1: How reliability depends on supplier reliability

- Suppose each of your suppliers is functional with probability r independently.
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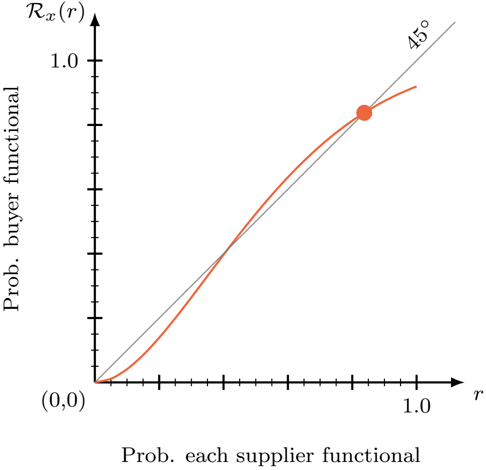
$$\mathcal{R}_x(r) = \left[\qquad (1 - xr) \quad \right]$$

Prob. given supplier is *not* available



Understanding the precipice

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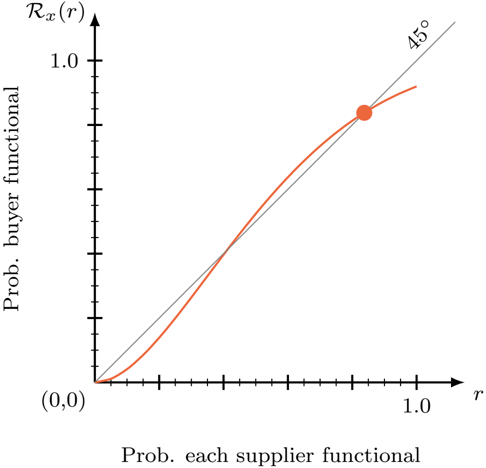
$$\mathcal{R}_x(r) = \left[\qquad (1 - xr)^{\,n} \right]$$

Prob. *all* suppliers of a given input not available



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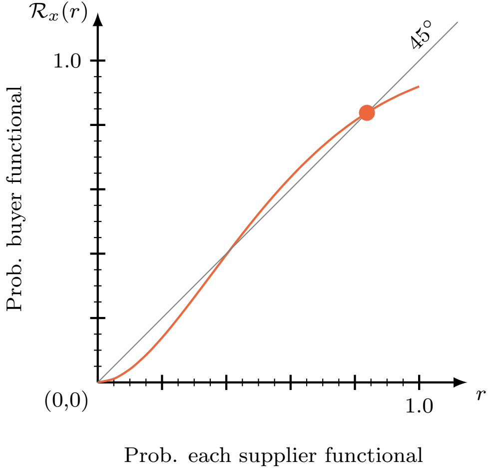
Prob. there is a supplier of a given input available



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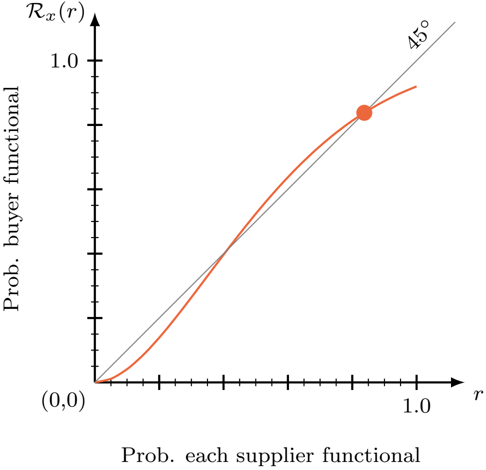
Prob. there is a supplier of *all* inputs available



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- Suppose each of your suppliers is functional with probability r independently.
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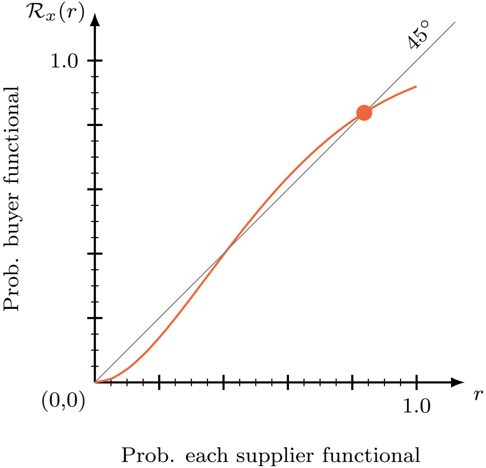
Fact. For $x \neq x_{\mathsf{crit}}$ the **largest fixed point** of $\mathcal{R}_x$ is equal to reliability as $\mu \to \infty$.



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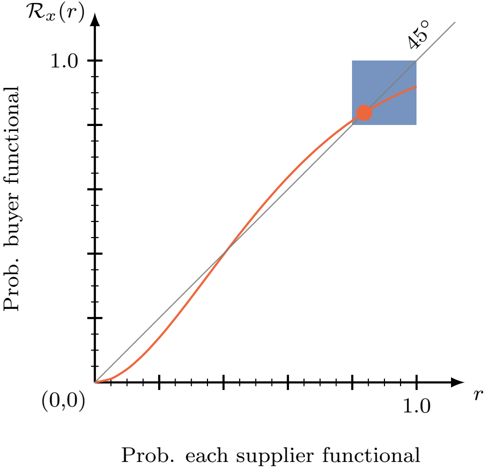
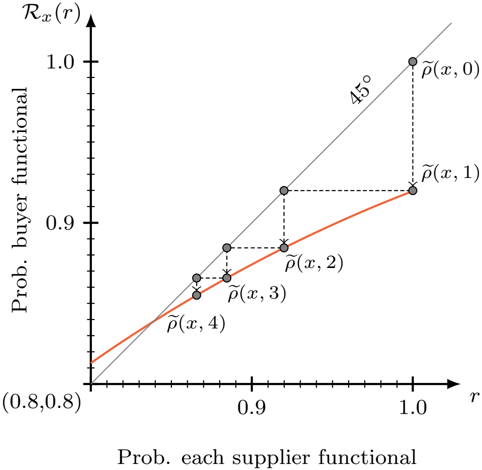
We will now zoom in on the area at the top right.



Understanding the precipice

Part 2: Computing reliability – an iteration

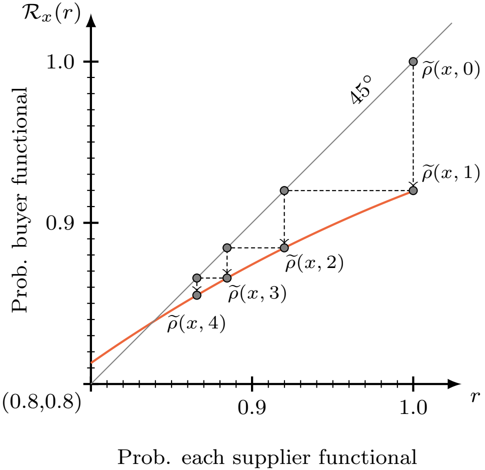
- Start with reliability of most upstream suppliers, $\widetilde{\rho}(x, 0) = 1$.



Understanding the precipice

Part 2: Computing reliability – an iteration

- Start with reliability of most upstream suppliers, $\widetilde{\rho}(x,0) = 1$.
- Plug that into $\mathcal{R}_x$ and call the output $\widetilde{\rho}(x,1) = \mathcal{R}_x(\widetilde{\rho}(x,0))$.



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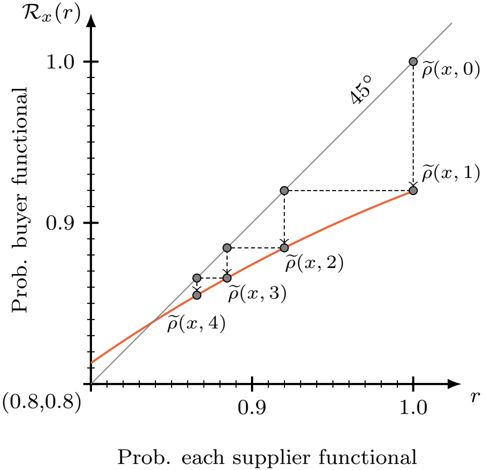
- Now continue iteratively.



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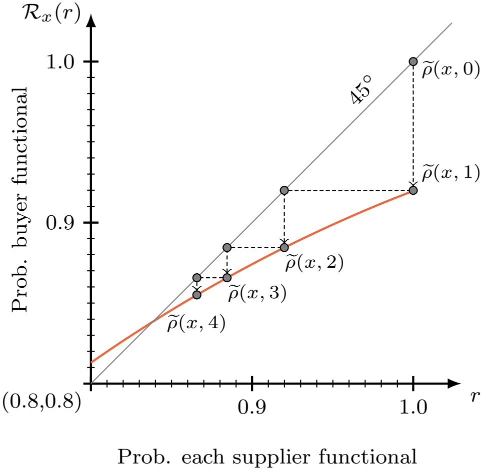
- Plug that into $\mathcal{R}_x$ and call the output $\widetilde{\rho}(x,1) = \mathcal{R}_x(\widetilde{\rho}(x,0))$.
- Now continue iteratively.
- Observe that it takes only a few steps to get quite close to the largest fixed point of $\mathcal{R}_x$.



Understanding the precipice

Part 3: How the largest fixed point depends on x

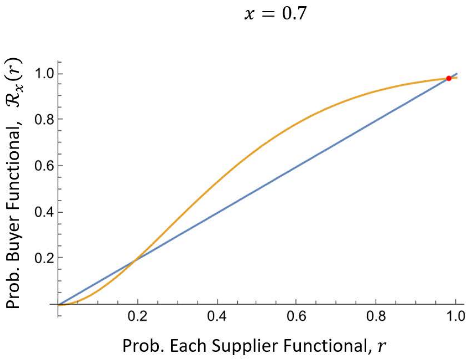
$x = 0.7$

Prob. Buyer Functional, $\mathcal{R}_x(r)$

Prob. Each Supplier Functional, r

Understanding the precipice

Part 3: How the largest fixed point depends on x



Understanding the precipice

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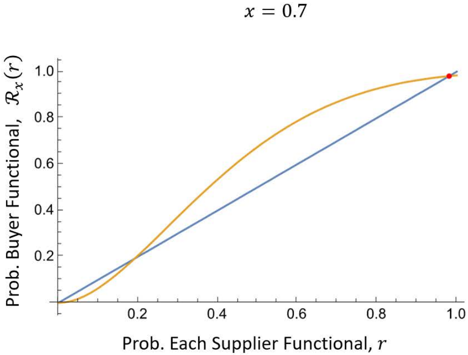

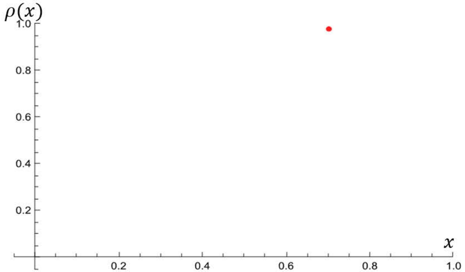

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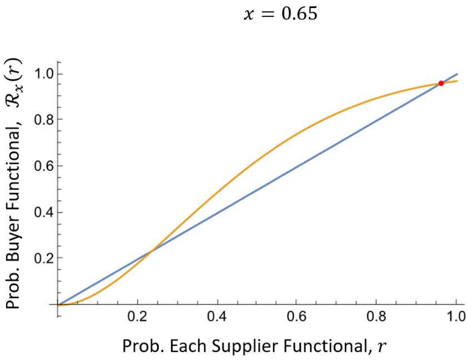

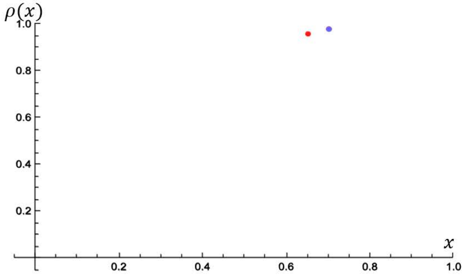

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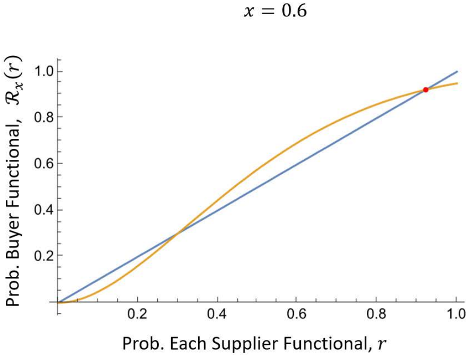

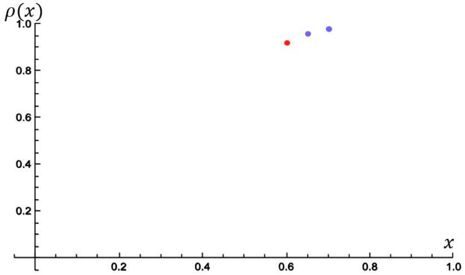

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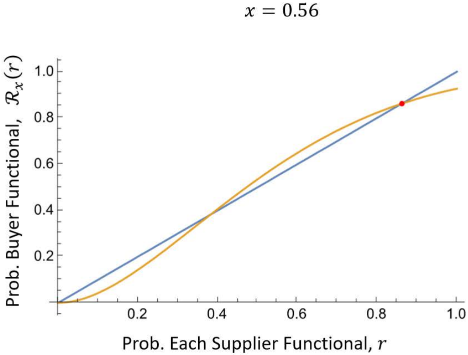

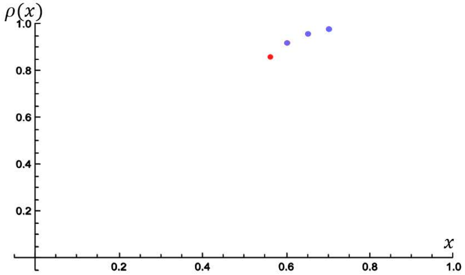

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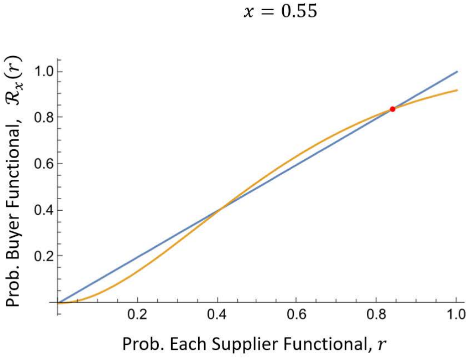

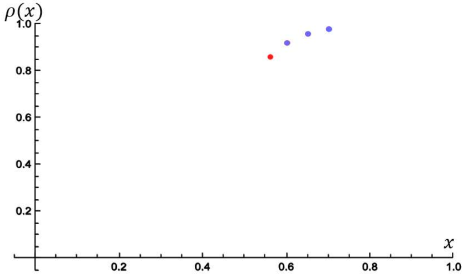

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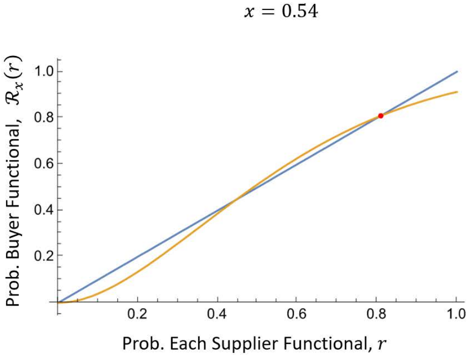

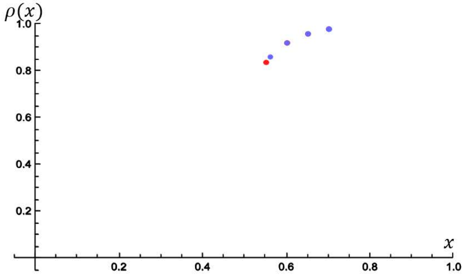

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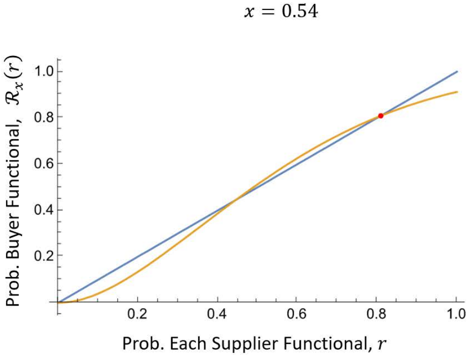

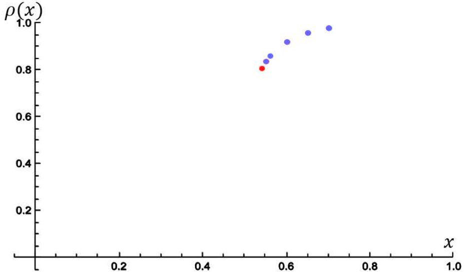

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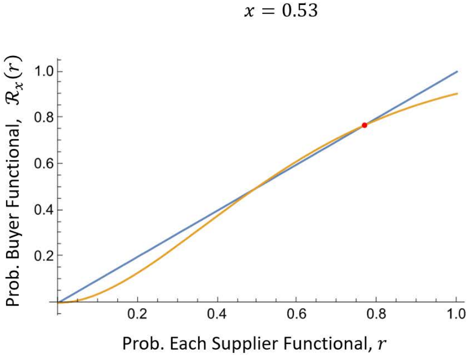

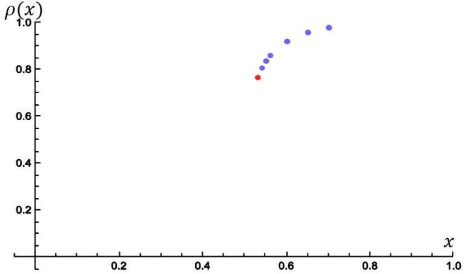

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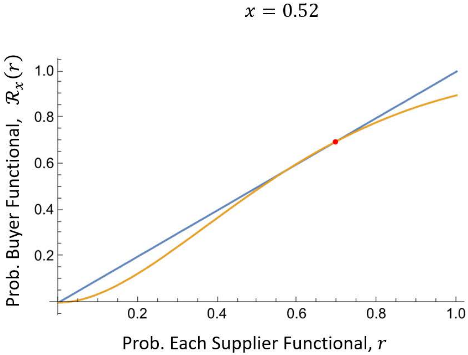

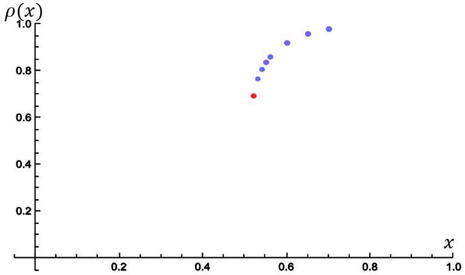

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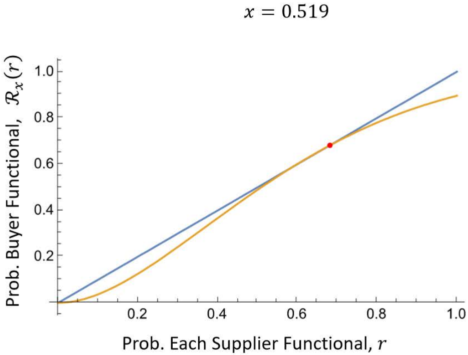

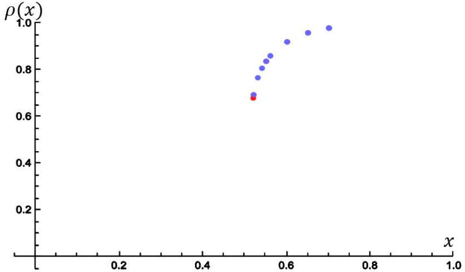

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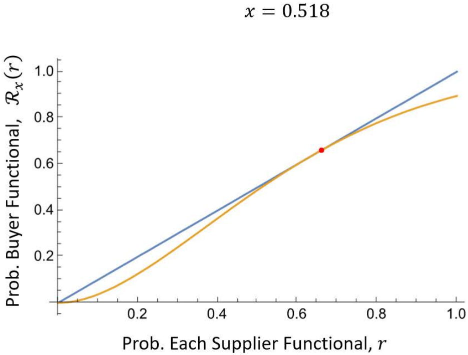

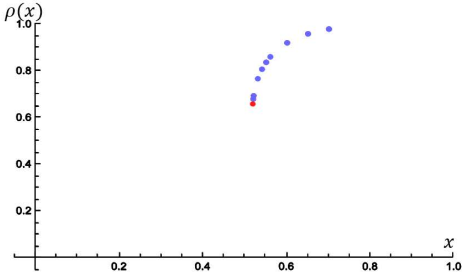

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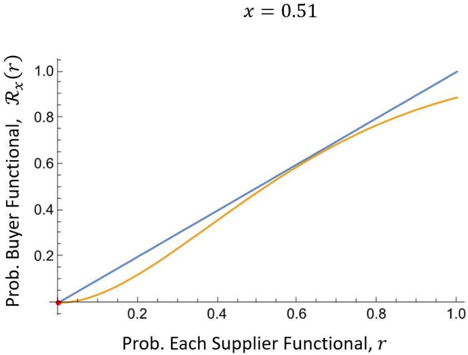

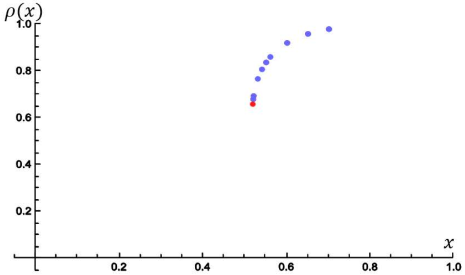

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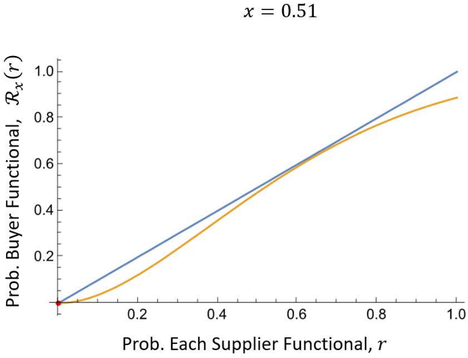

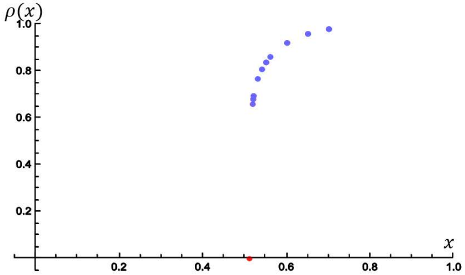

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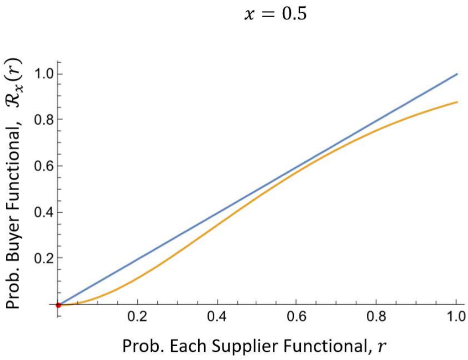

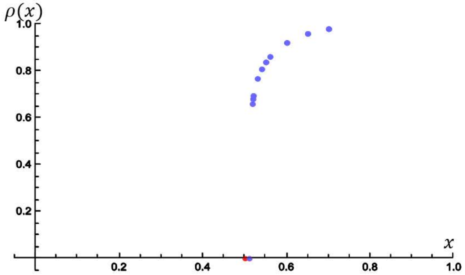

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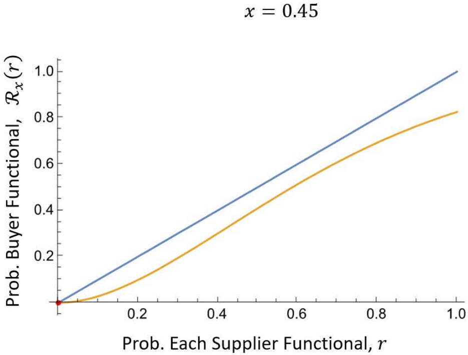

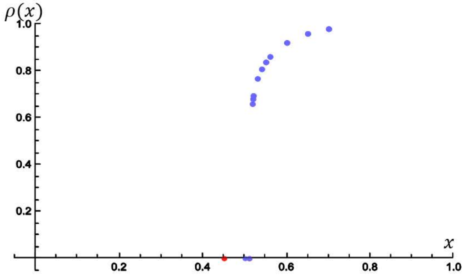

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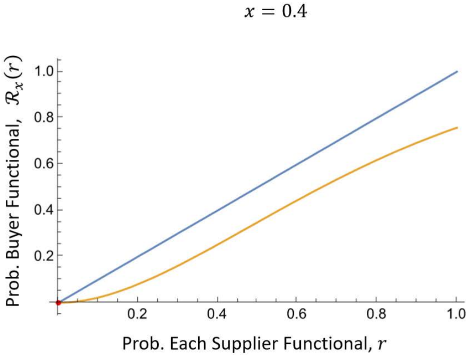

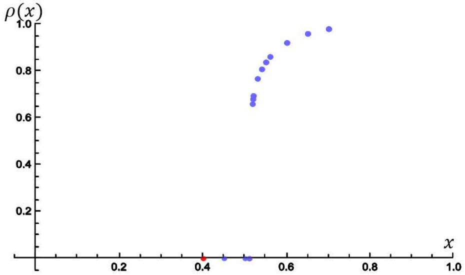

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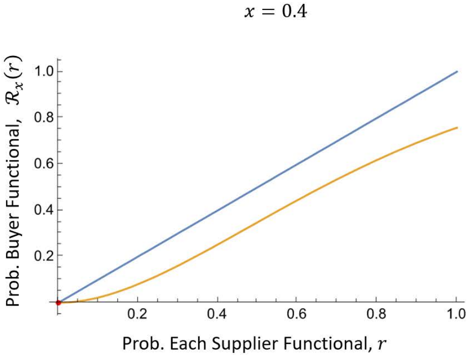
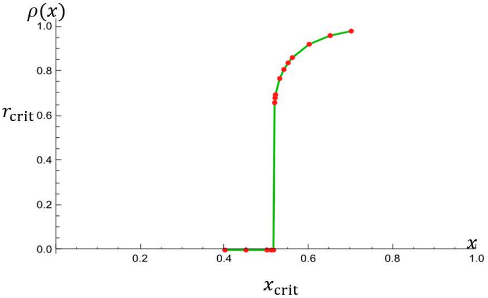
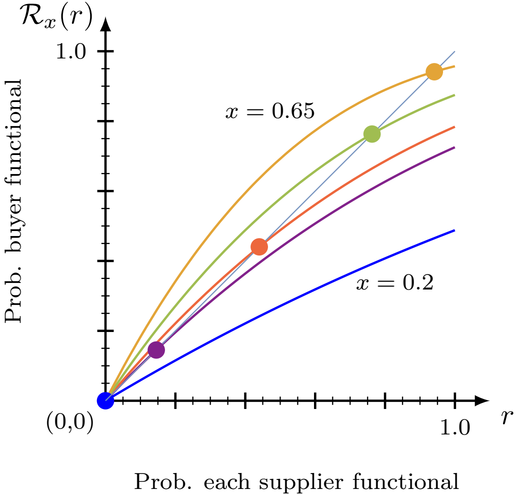




Contrast with simple production: One input per step, No precipice



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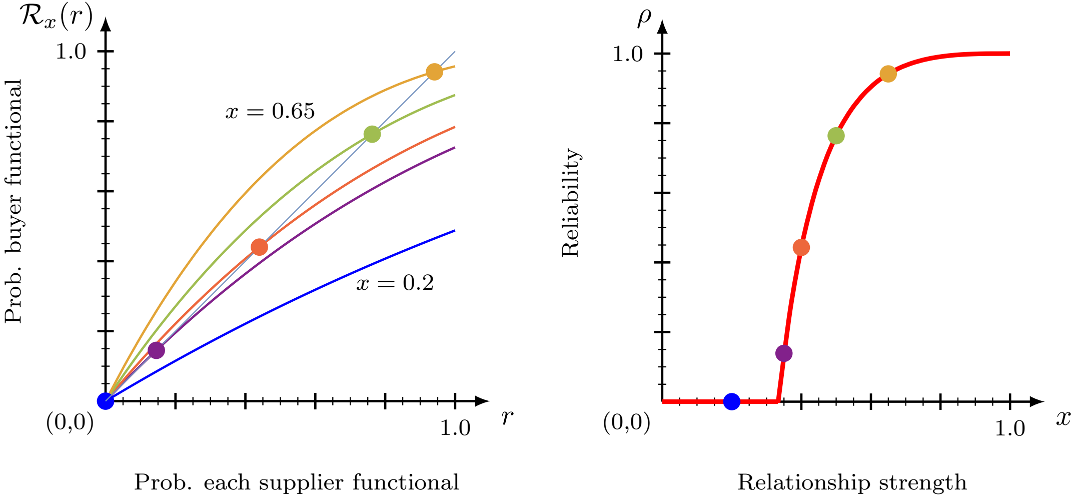

Network formation: Endogenous choice of x

MODEL

We now let each firm choose the strength of its links. (Interpretation: investing in relational contracts.)

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We now let each firm choose the strength of its links. (Interpretation: investing in relational contracts.)

- Simultaneously, each firm chooses a non-contractable relationship strength investment x_{if} before supply network is realized.

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- All of the firm's links work with probability x_{if}.

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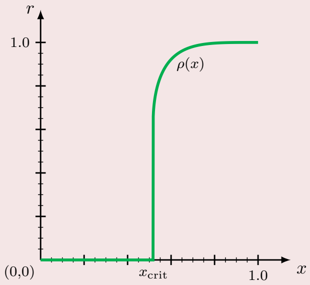
More

QUESTION AND KEY NOTIONS



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▸ More

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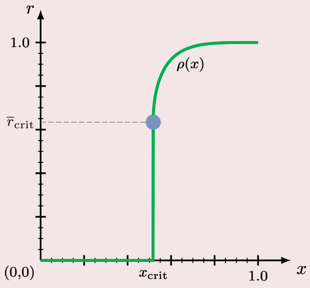

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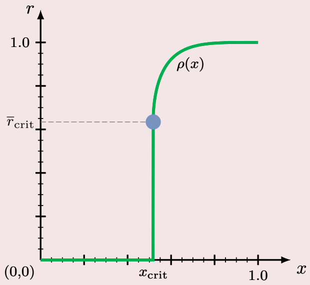
More

QUESTION AND KEY NOTIONS



If relationship strengths are chosen efficiently, will the supply network be on the precipice?

Network formation: Endogenous choice of x

MODEL

We now let each firm choose the strength of its links. (Interpretation: investing in relational contracts.)

- Simultaneously, each firm chooses a non-contractable relationship strength investment x_{if} before supply network is realized.
- All of the firm's links work with probability x_{if}.
- This costs $c(x_{if})$ up front, where c satisfies standard conditions.

If the firm produces, it earns gross profit $\kappa g(\rho_i)$, where

- ρ_i is competitor reliability and
- g is decreasing
- κ is a productivity shifter

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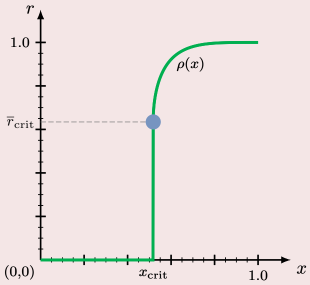

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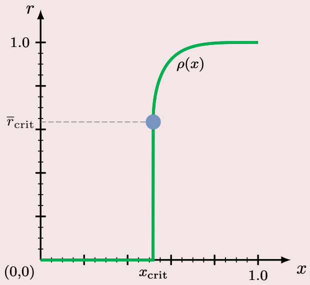

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When relationship strengths are chosen in equilibrium, will the supply network be on the precipice?

Equilibrium definition

For a given κ, we say an outcome $x \in [0,1]$ is a symmetric undominated equilibrium (SUE) if

- **[each firm is optimizing]**: for gross profits $\kappa g(\rho)$, the investment level $x_{if} = x$ for all firms if is a Nash equilibrium of the investment game . . .

$$\left(\text{i.e., } x \text{ maximizes } \kappa g(\rho) P_{i \text{ functions}}(x_{if}; x) - c(x_{if}) \right)$$

- . . . that maximizes total surplus among the symmetric Nash equilibria. **[efficient selection]**

We will focus on these.

Network formation: Endogenous choice of x

INVESTMENT EQUILIBRIUM ANALYSIS

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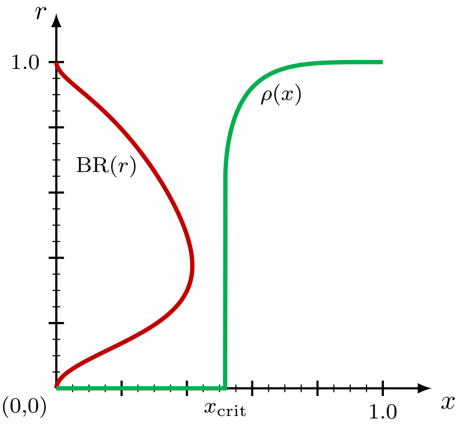

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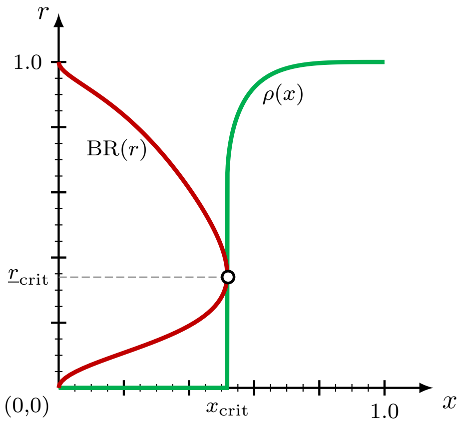

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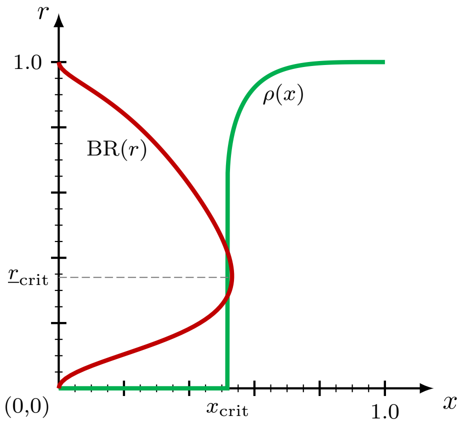

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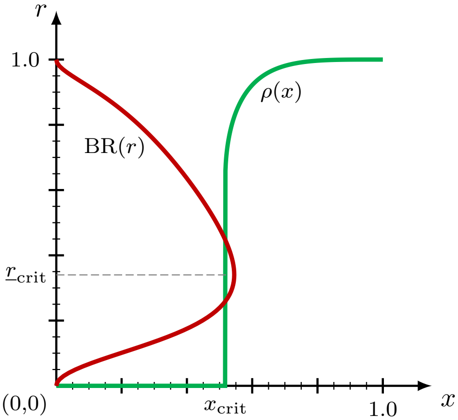

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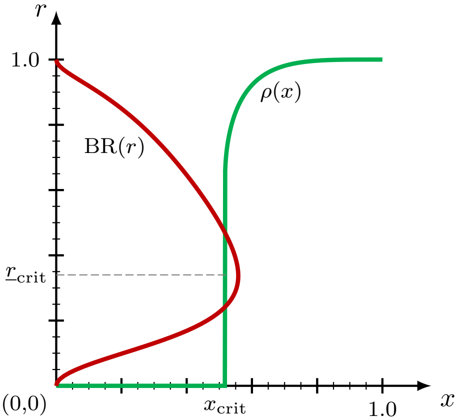

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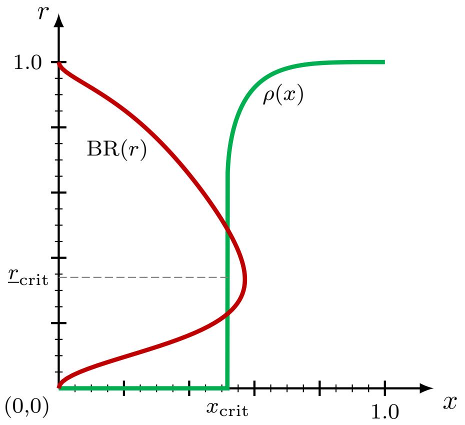

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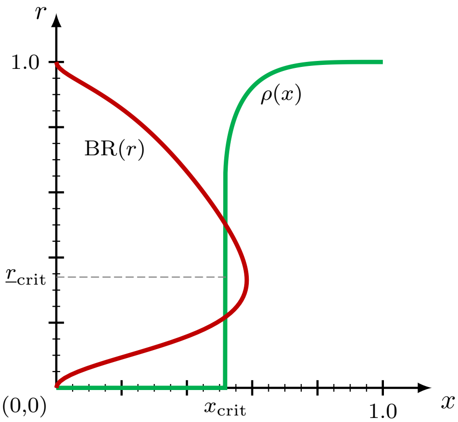

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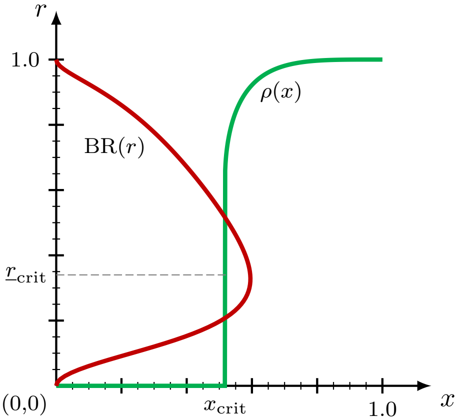

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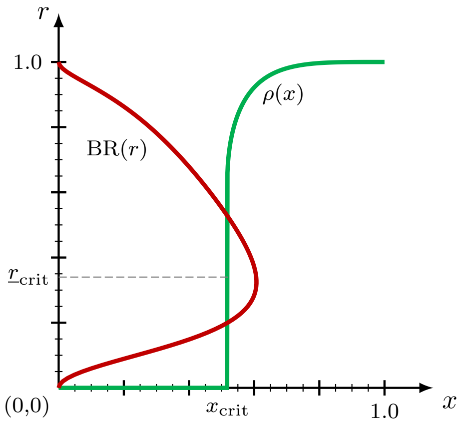

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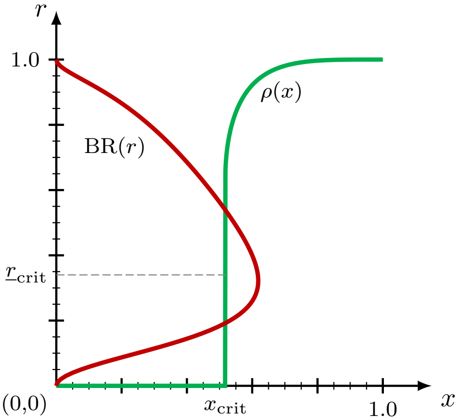

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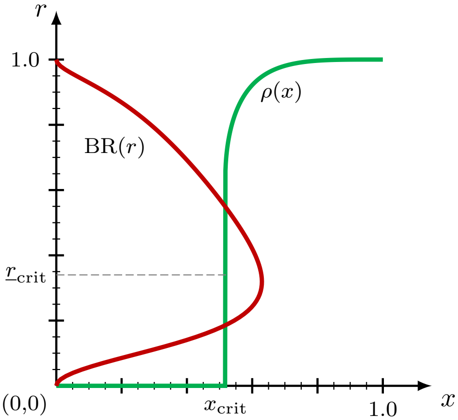

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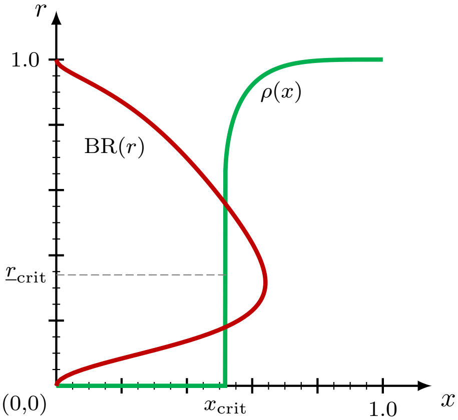

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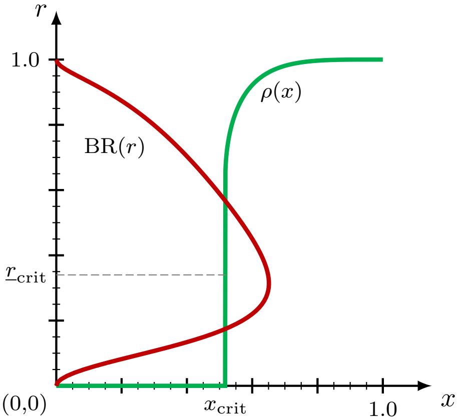

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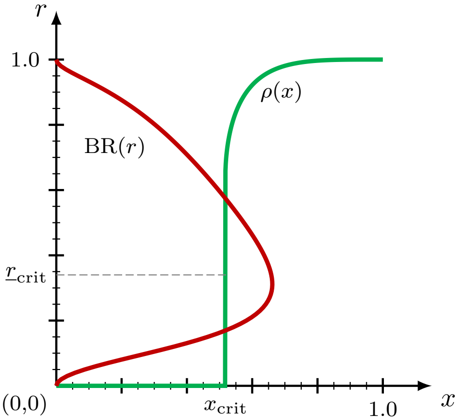

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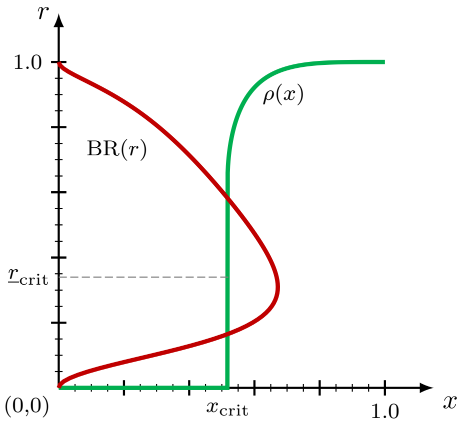

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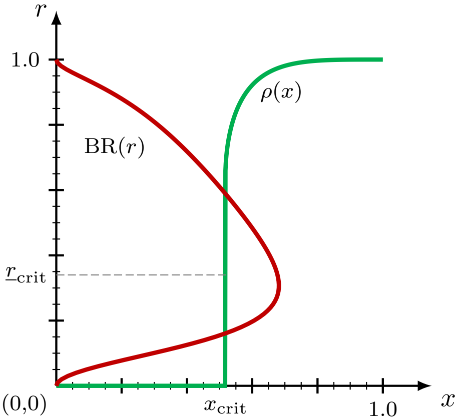

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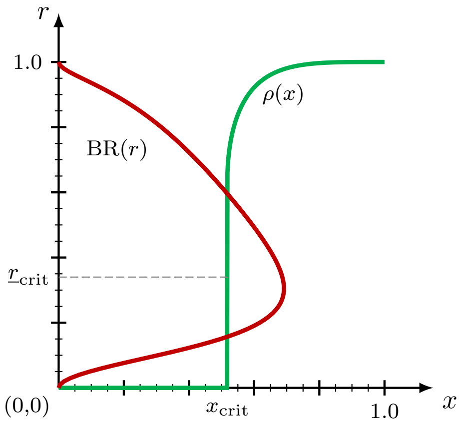

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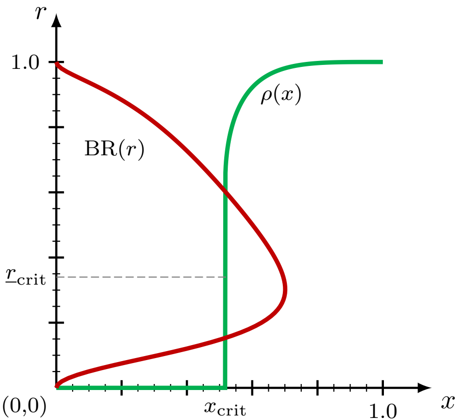

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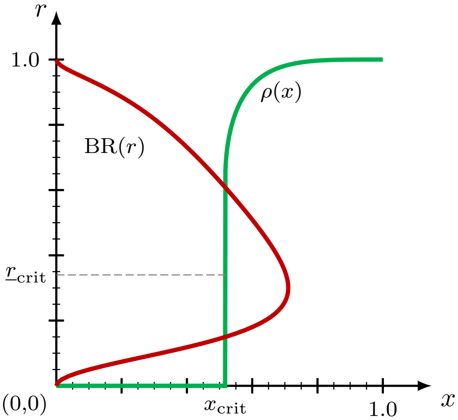

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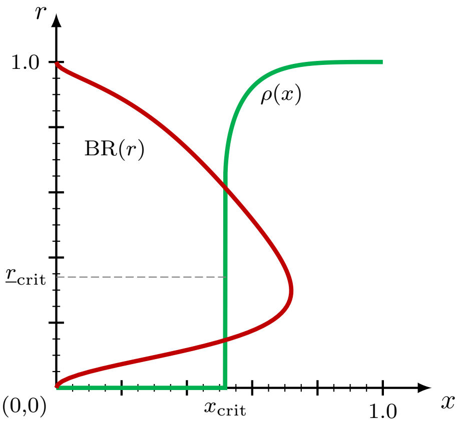

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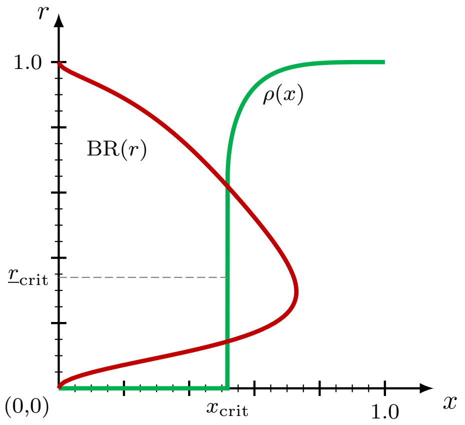

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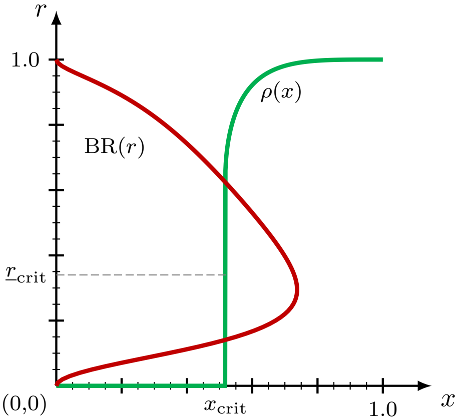
When supply networks are sufficiently deep, there is an interval $[\underline{\kappa}, \bar{\kappa}]$ so that for κ in that interval, the (undominated) equilibrium x is arbitrarily close to the precipice.

For κ below the interval, equilibrium is unproductive.



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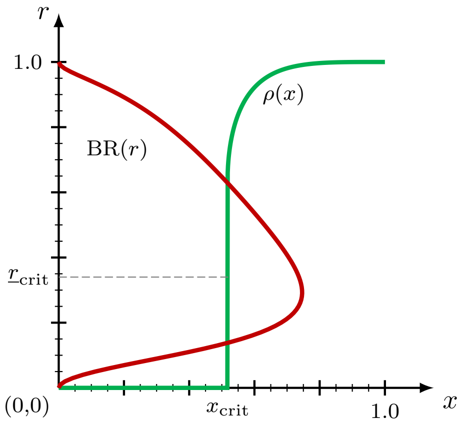

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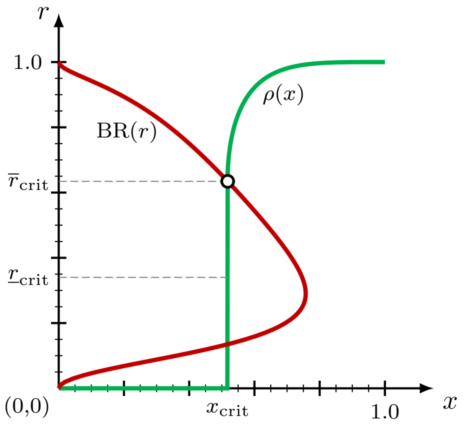

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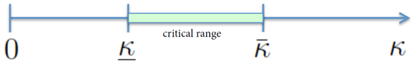

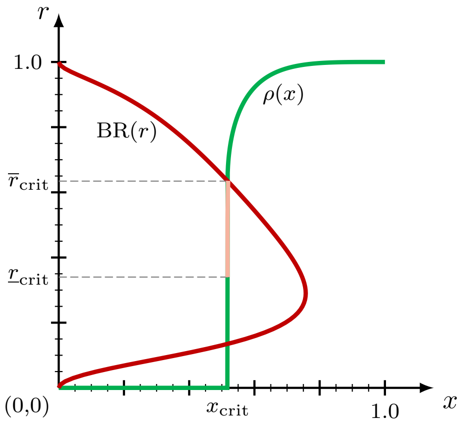

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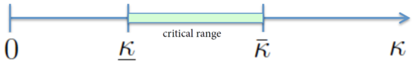

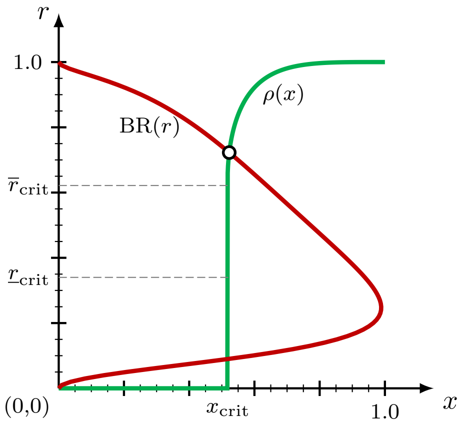

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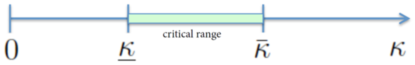

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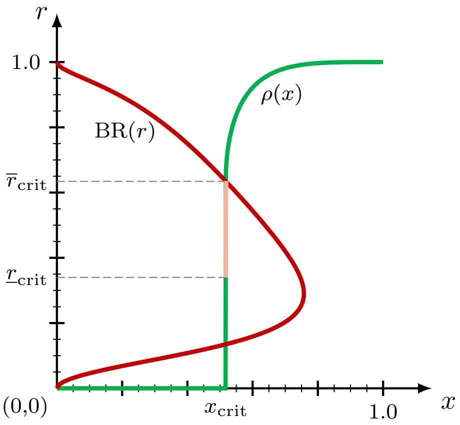

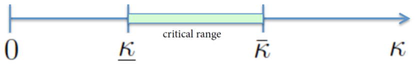

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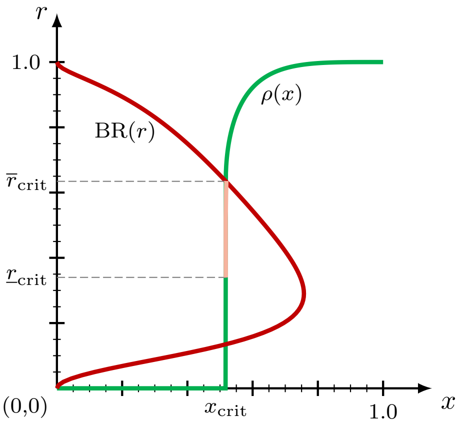

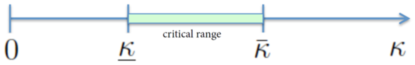

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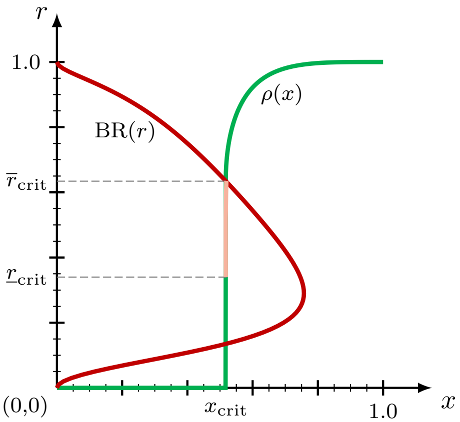
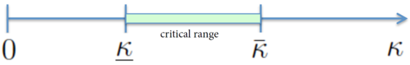
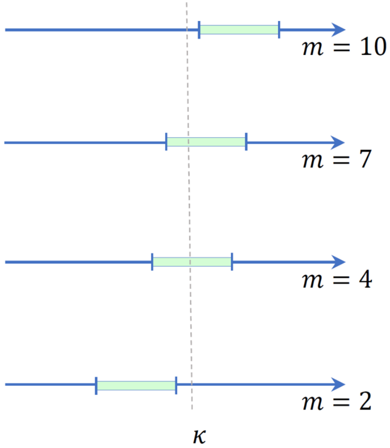
An arbitrarily small shock reducing all relationship strengths from x to $x - \varepsilon$ causes reliability to collapse.





Varying the complexity of production

A THOUGHT EXPERIMENT



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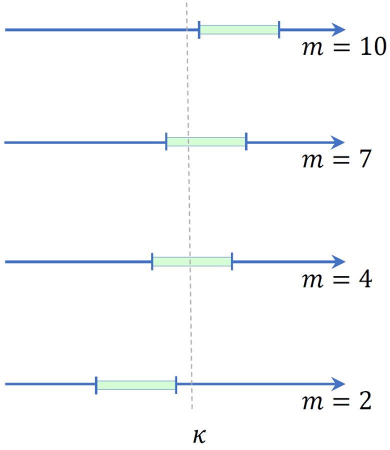
Hold everything in the model fixed, and vary m, the number of inputs required by each firm.



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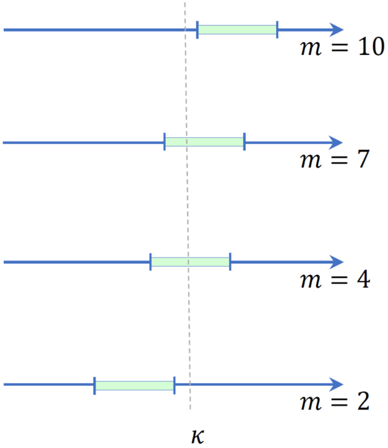
Result: both $\underline{\kappa}$ and $\bar{\kappa}$ are increasing in m.



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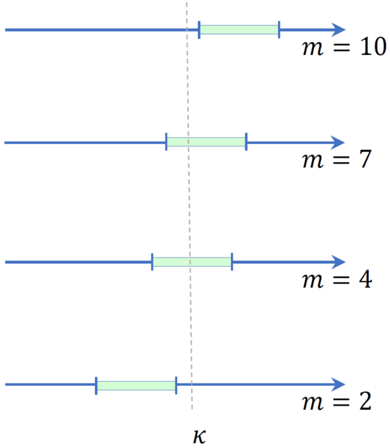

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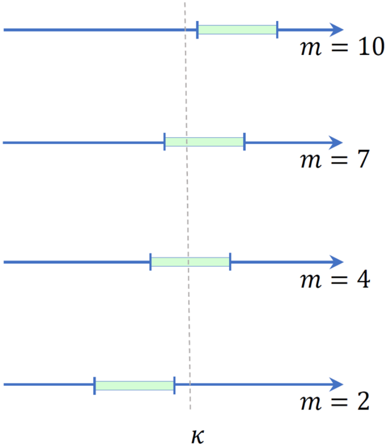
Ones of intermediate complexity can be made, but are susceptible to fragility.



Varying the complexity of production

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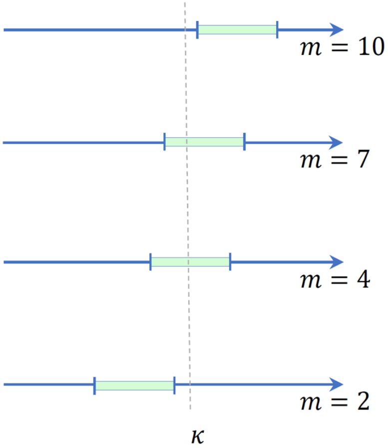

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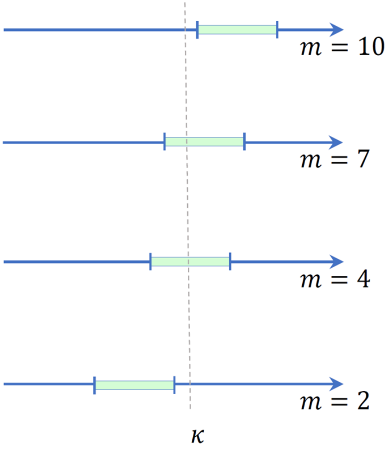

Concluding thoughts

WHAT'S THE MARKET FAILURE?

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Companion piece: Elliott and Golub "Networks and Economic Fragility" (ARE 2022). Underlying facts, survey of "extensive margin" forces in production networks.

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Back

Related Literature

Network formation theory, reliability, and risk: e.g., Bala and Goyal (2000), Levine (2012), Goyal and Vigier (2014), Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), Brummitt et al. (2017), Elliott, Georg and Hazell (2018), Erol (2018), Erol and Vohra (2018), Talamàs and Vohra (2018), Bimpikis, Candogan, and Ehsan (2019), Dasaratha (2020).

> Our contribution: A tractable network formation model for large complex supply networks with new features.

Complementarities in production and their implications: e.g., Kremer (1993), Blanchard and Kremer (1997), Ciccone (2002), Acemoglu, Antràs and Helpman (2007), Angeletos and Pavan (2007), Jones (2011), ...

> Our contribution: Possible concern—might actions that mitigate supply risks endogenously dampen the complementarities. We show they don't.

Related Literature

Production networks: e.g., Long and Plosser (1983), Horvath (1998), Dupor (1999), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011), Elliott, Golub, and Jackson (2014), ... Taschereau-Dumouchel (2017), Boehm and Oberfield (2018) and König et al. (2019), Baqaee and Farhi (2019, 2020), Acemoglu and Tahbaz-Salehi (2020).

> Our contribution: This literature focuses on smooth nonlinearities. We show how sourcing failures at the micro level give rise to discontinuities.

Self-organizing criticality and phase transitions: Jovanovic (1987), Scheinkman and Woodford (1994), an engineering/math lit. e.g., Buldyrev et al. (2010), Tang et al. (2016), and Yang et al. (2019).

> Our contribution: Fully microfounded model that shows that the most severe phase transition occurs in the most classical production network setting (once you have our kinds of failures). In our setting economy is robust to idiosyncratic shocks.

Back

Examples of idiosyncratic disruptions

Fire at Philips Semiconductor halted production, preventing Ericsson from sourcing critical inputs, causing its production to also stop. Ericsson lost $> \$100M$ in sales, subsequently exited mobile phone business (Norrman and Jansson, 2004).

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"It is tempting to think of supply chain disruptions as rare events. However, although a given type of disruption (earthquake, fire, strike) may occur very infrequently, the large number of possible disruption causes, coupled with the vast scale of modern supply chains, makes the likelihood that some disruption will strike a given supply chain in a given year quite high." (Supply Chain Quarterly, 2018)

Resilinc found 1,069 supply chain disruption events globally during a six-month period in 2018.

Why the Pandemic Has Disrupted Supply Chains

JUNE 17, 2021 • ARTICLES

By Susan Helper and Evan Soltas

Figure 1. Businesses Have Little Inventory to Sell

Inventory-to-sales ratio (days of sales in inventory)

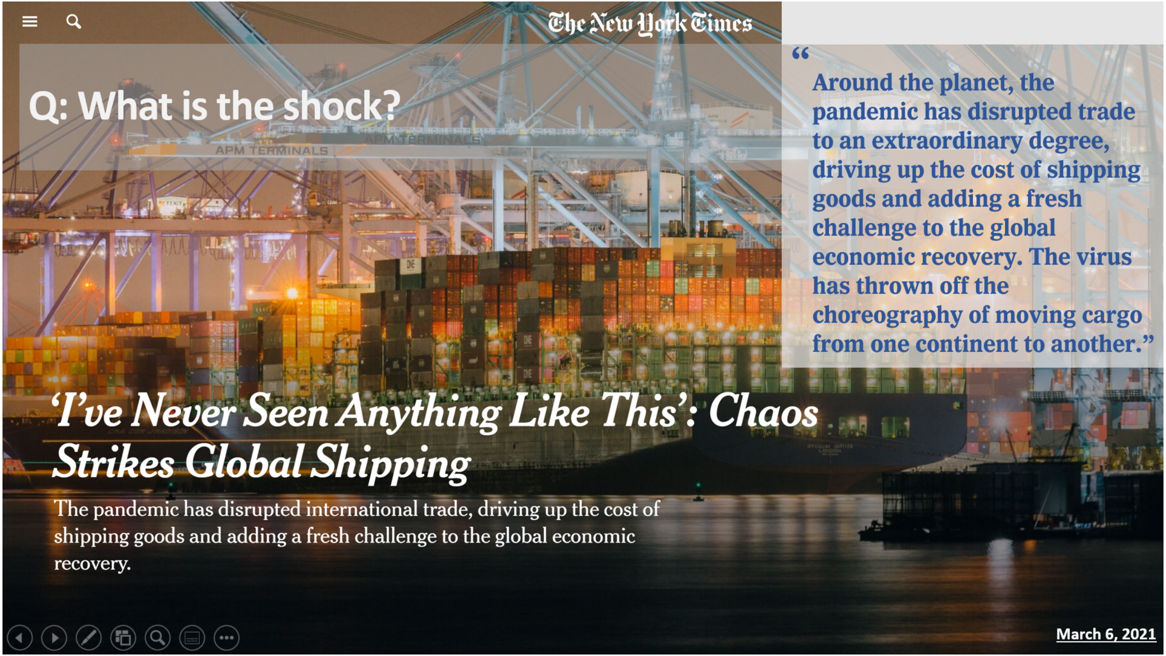
Sources: U.S. Census Bureau; CEA Calculations

Figure 2. Supply-Chain Disruptions By Sector

In the last week, did this business have domestic supplier delays? (percentage saying yes)

Sources: U.S. Census Bureau; CEA Calculations.

Council of Economic Advisers



The New York Times

Q: What is the shock?

"Around the planet, the pandemic has disrupted trade to an extraordinary degree, driving up the cost of shipping goods and adding a fresh challenge to the global economic recovery. The virus has thrown off the choreography of moving cargo from one continent to another."

'I've Never Seen Anything Like This': Chaos Strikes Global Shipping

The pandemic has disrupted international trade, driving up the cost of shipping goods and adding a fresh challenge to the global economic recovery.

March 6, 2021

Six months ago, [a manufacturer] was paying about $2,500 to ship a 40-foot container to California.“We just paid $6,000 to $7,000,” he said. “This is the highest freight rate that I have seen in 45 years in the business.”

In early September, he waited 90 days to secure space on a ship for a container of wicker chairs and tables.

“I’ve never seen anything like this,” said Lars Mikael Jensen, head of Global Ocean Network at A.P. Moller-Maersk, the world’s largest shipping company. “**All the links in the supply chain are stretched.** The ships, the trucks, the warehouses.”

Pandemic shocks have led to many disruptions in the details of shipping.

- mismatch between containers and ships (emergency shipping left containers in places where ships rarely are);

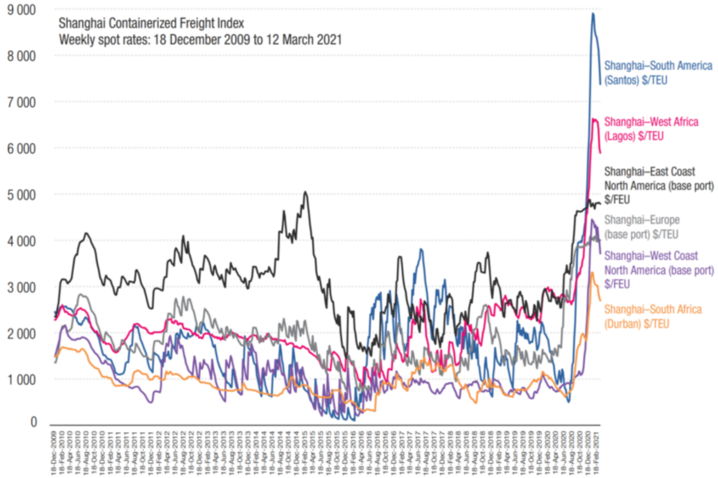
  adds 1000-mile Kolkata-Mumbai leg

- slowdowns at borders due to limited staffing (quarantines, childcare);

  congested ports

- Containers going Eastward empty because of urgency.

Shipping costs have spiked dramatically



Abbreviations: FEU, 40-foot equivalent; TEU, 20-foot equivalent.
Source: UNCTAD calculations, based on data provided by Clarksons Research.

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Proposition

Suppose all complexities $m_i \geq 2$. Let $x_{if,j} = X_{ij}(\xi)$, where $X_{ij} : [0,1] \to [0,1]$ is a strictly increasing C^1 onto function and ξ is an economywide parameter (e.g., institutional quality). There is a critical ξ_{crit} such that $\lim_{\tau\to\infty} \boldsymbol{\rho}(\xi,\infty) = 0$ for all $\xi < \xi_{\text{crit}}$ and $\lim_{\tau\to\infty} \boldsymbol{\rho}(\xi,\infty) > r_{\text{crit}} > 0$ for all $\xi > \xi_{\text{crit}}$.

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