Expectations, Networks, and Conventions

Benjamin Golub
Harvard

Stephen Morris
Princeton

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Consider a situation where agents care about matching two targets—
uncertainty about both:

others' actions; a "fundamentally" best action.

Conventions (in organizations, choice of language, speculative trading…):
actions selected in equilibrium when coordination is important.

Question: How do conventions depend on differences in

(i) information (signals) (ii) interpretation (priors) (iii) coordination concerns (interaction)

beliefs and higher-order beliefs networks

Contribution: Analyze effects of (i), (ii), (iii) together via reduction of all three to a network. Yields **unification** and **new purely informational results**.

MODEL

Agents	$i \in N$
External state	$\theta \in \Theta$
i's fundamental	$y^i : \Theta \to [-M, M]$
i's types	$t^i \in T^i$
belief f'n.	$\pi^i : T^i \to \Delta(\Theta \times T^{-i})$
	$\rightsquigarrow E^i$ i's expectation
strategy	$a^i : T^i \to [-M, M]$

ex post payoff

$$u^i = -\overbrace{\beta}^{\in(0,1)} \sum_j \gamma^{ij} (a^i - a^j)^2 - (1-\beta)\,(a^i - y^i(\theta))^2$$

$$BR^i = \underbrace{\beta \sum_{j \neq i} \gamma^{ij} E^i a^j}_{\text{matching others' actions}} + \underbrace{(1-\beta)\, E^i y^i}_{\text{matching fundamental}}$$

Ex. Net Game, Complete Info.

$$u^i = -\beta \sum_j \gamma^{ij} (a^i - a^j)^2 - (1-\beta)(a^i - y^i)^2$$

$$\begin{tikzcd}
y^1 \arrow[r, bend left, "1"] & y^2 \arrow[l, bend left, "\gamma^{21}"] \arrow[r, bend right, "\gamma^{23}"'] & y^3 \arrow[l, bend right, "1"']
\end{tikzcd}$$

Ex 2 agents, incomplete info

$$u^i = -\beta (a^i - a^j)^2 - (1-\beta)(a^i - y(\theta))^2$$

$\theta \in \{G, B\}$

$\rho^i \in \Delta(\Theta)$ i's prior

$t^i \in \{g^i, b^i\}$ matches θ w.p. q^i

π^i computed via Bayes' rule

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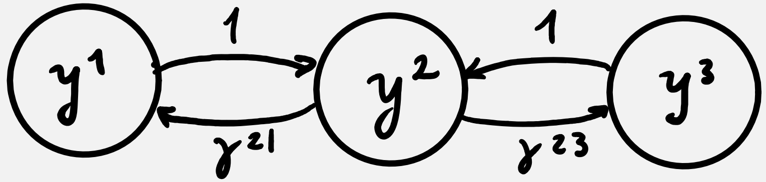
Ex. Net Game, Complete Info.

$$-u^i = \beta \sum_j \gamma^{ij} (a^i - a^j)^2 + (1-\beta)(a^i - y^i)^2$$



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Fact 1 The game has a unique rationalizable strategy profile.

Question: How does play depend on (i) information; (ii) priors (iii) network ?

Focus: Conventions: play as $\beta \uparrow 1$.

Ex 2 agents, incomplete info

$$-u^i = \beta(a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2$$

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$t^i \in \{g^i, b^i\}$ matches θ w.p. q^i

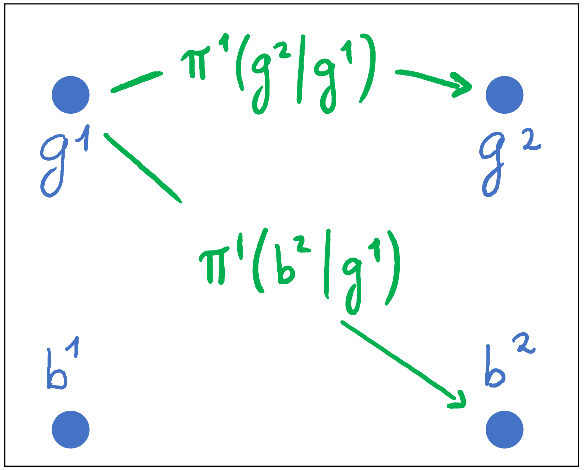
π^i computed via Bayes' rule

KEY IDEA: Incomplete-info. aspect can be reduced to network aspect

KEY DEVICE: "interaction structure"

nodes: $S = \bigcup_i T^i$

edges: $B(t^i, t^j) = \gamma^{ij} \pi^i (t^j | t^i)$



Define

$$f(t^i) = E^i[y^i | t^i].$$

FACT 2 The unique rationalizable action profile is given by

$$a = (1-\beta)(I - \beta B)^{-1} f$$

FACT 1 The game has a unique rationalizable strategy profile.

QUESTION: How does play depend on (i) information; (ii) priors (iii) network ?

FOCUS: Conventions: play as $\beta \uparrow 1$.

Ex 2 agents, incomplete info

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KEY DEVICE: "interaction structure"

nodes: $S = \bigcup_i T^i$

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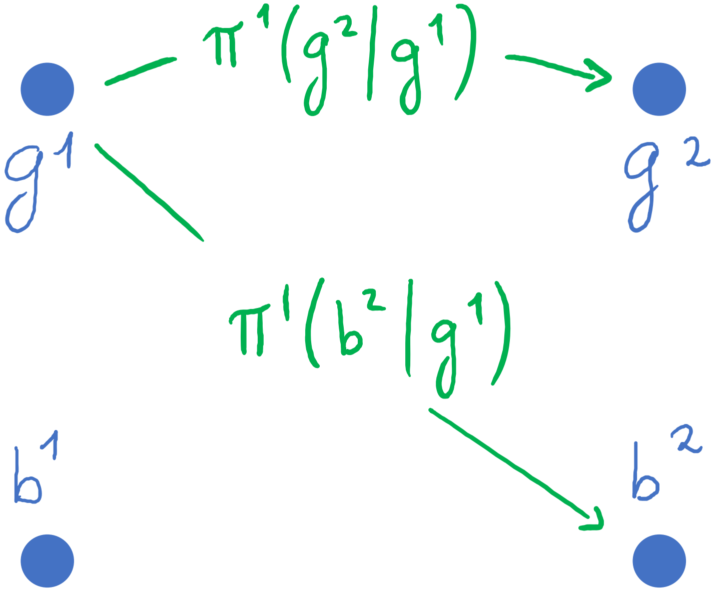
Shin and Williamson (*GEB* 96) "How Much Common Belief is Necessary for a Convention?"

Morris (1997) "Interaction Games"
Morris (*REStud* 2000) "Contagion"



PROP 1 $c(\vec{y};\vec{\pi},\vec{\Gamma}) = \sum_{t^i \in S} p(t^i) f(t^i)$

where p is unique $p \in \Delta(S)$ s.t. $pB = p$

i.e. p is the stationary distribution of B, viewed as a Markov chain.

PROP 0: If B str. connected, then as $\beta \uparrow 1$,

$\forall i \; a^i(t^i) \to c(\vec{y};\vec{\pi},\Gamma)$: "the convention"

KEY IDEA: Incomplete-info. aspect can be reduced to network aspect
→ analyze how info. struct. matters.

KEY DEVICE: "interaction structure"

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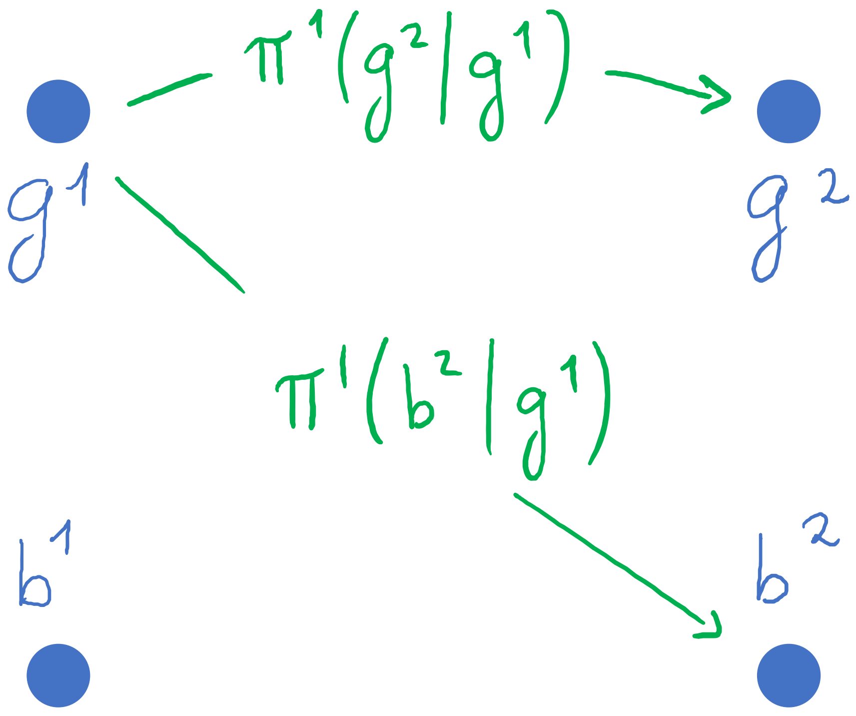

APPLICATIONS

1. Contagion of Optimism

2. (Pseudo) Common Prior
   Influence $\propto$ net centrality

3. Tyranny of least-informed

Key Device: "interaction structure"

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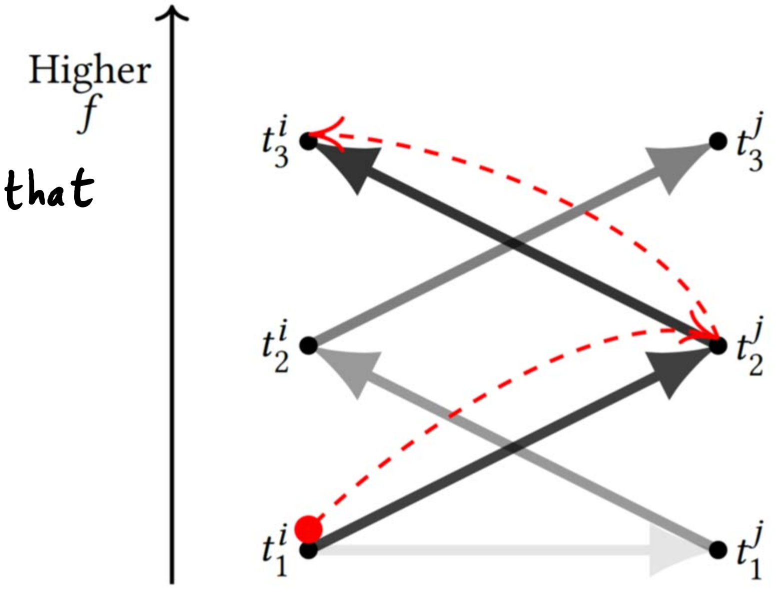
Contagion of Optimism

Suppose each i is certain each counterparty has $E^j y \geq E^i y + \delta$, unless $E^i y \geq \bar{f}$; in that case, $E^j y \geq E^i y$.

Then $c(y; \vec{\pi}, \Gamma) \geq \bar{f}$

Reason: for t^i s.t. $f(t^i) < \bar{f}$, the B process can only move upward.



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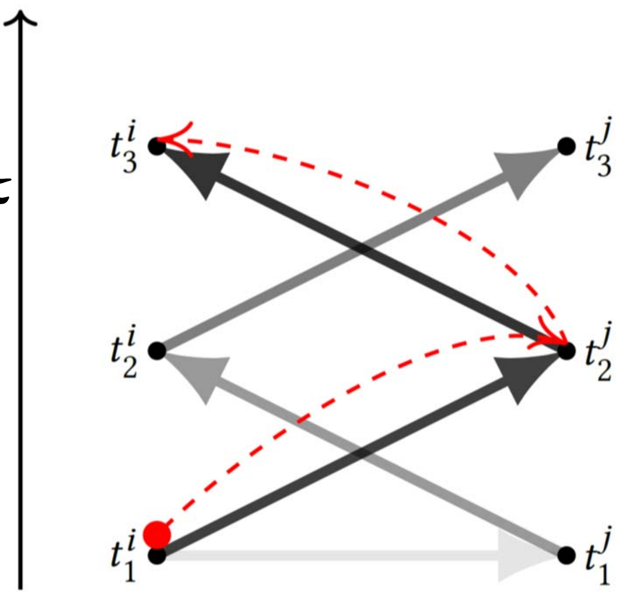
Contagion of Optimism

Suppose each i is second-order optimistic (on avg)

$\underbrace{\sum_j \gamma^{ij} E^i E^j y} \geq E^i y + \delta$, unless $E^i y \geq \bar{f}$; in that case, " " $\geq E^i y - \varepsilon$.

Then $c(y; \vec{\pi}, \Gamma) \geq \bar{f} / (1 + \varepsilon/\delta)$



Reason: for t^i s.t. $f(t^i) < \bar{f}$, B process moves upward on average

Harrison and Kreps (*QJE* 1978) “Speculative Investor Behavior…”

Izmalkov and Yildiz (*AEJ:Micro* 2010) “Investor Sentiments”

Han and Kyle (*MS* 2017) “Speculative Equilibrium with Differences in Higher-Order Beliefs”

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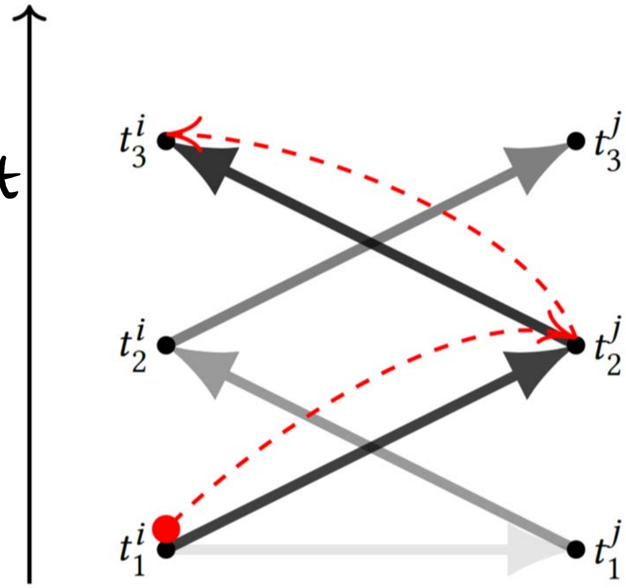

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Then $c(y; \vec{\pi}, \Gamma) \geq \bar{f} / (1 + \varepsilon/\delta)$

Reason: for t^i s.t. $f(t^i) < \bar{f}$, the B process moves upward on average

Proof: Take MC $W_0, W_1, \ldots,$ with ergodic dist p. Suppose $\exists\ \delta, \varepsilon$ s.t.

$$f(s) < \bar{f} \Rightarrow \mathbb{E}_{W_0 = s}[f(W_1)] \geq f(s) + \delta$$

$$f(s) \geq \bar{f} \Rightarrow \mathbb{E}_{W_0 = s}[f(W_1)] \geq f(s) - \varepsilon$$

Then $p(s : f(s) \geq \bar{f}) \geq \dfrac{1}{1 + \varepsilon/\delta}$

Follows from

$$\mathbb{E}_{W_0 \sim p}[W_1 - W_0] = 0$$

Higher-Order Average Expectations

$$x^i_{t^i}(1) = E^i[y^i \mid t^i]$$

1st-order expectation of y^i given i's info

$$x^i_{t^i}(n+1) = \sum_j \gamma^{ij} E^i[x^j(n) \mid t^i]$$

$(n+1)^{th}$-order avg. expectation
an average of n^{th}-order exp. given i's info

Relation to Game

$$[B^n f](t^i) = x^i_{t^i}(n+1)$$

$$a_{eqm} = (1-\beta)(I - \beta B)^{-1} f = (1-\beta) \sum_{n=0}^{\infty} \beta^n B^n f$$

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Samet (*JET* 98) "Iterated Expectation and Common Priors"

Our companion paper: "Higher-Order Expectations"

$$a^i_{eqm}(t^i) = (1-\beta) \sum_{n=0}^{\infty} \beta^n x^i_{t^i}(n+1)$$

Common Priors & Influence

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where p is unique $p \in \Delta(S)$ s.t. $pB = p$

I.e. p is the stationary distribution of B, viewed as a Markov chain.

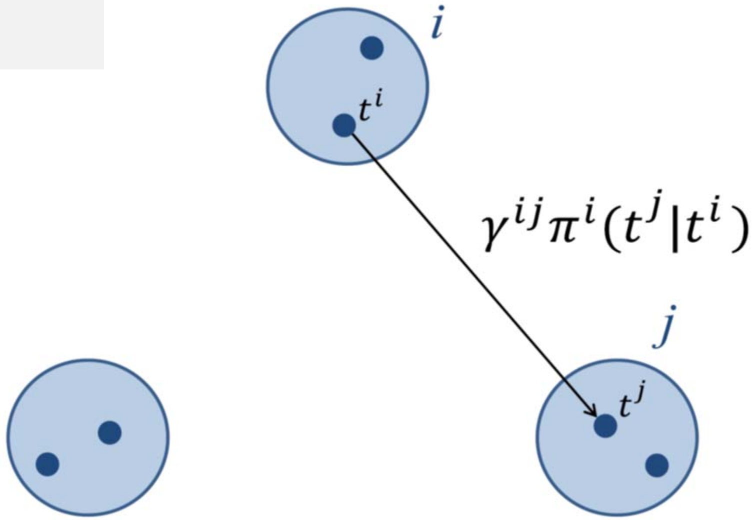
Lemma $\forall i$

$$\sum_i p(t^i) = e^i$$

Def $e(\Gamma)$ is defined as unique $e \in \Delta(N)$ s.t. $e\Gamma = e$.

Prop 2 common prior $\Rightarrow$ $c = \sum_i e^i \underbrace{\mathbb{E}\, y^i}_{}$

where $\mathbb{E}\, y^i = \sum_{t^i \in T^i} \mu(t^i)\, E^i[y^i \mid t^i]$



Common Priors & Influence

Def common priors over signals (CPS)

π^i all compatible w/ a $\hat{\mu} \in \Delta(T)$

$\Downarrow$

$\exists$ priors $(\mu^i)_{i \in N}$ s.t.

$$\mu^i(t^i)\pi^i(t^j \mid t^i) = \mu^j(t^j)\pi^j(t^i \mid t^j)$$

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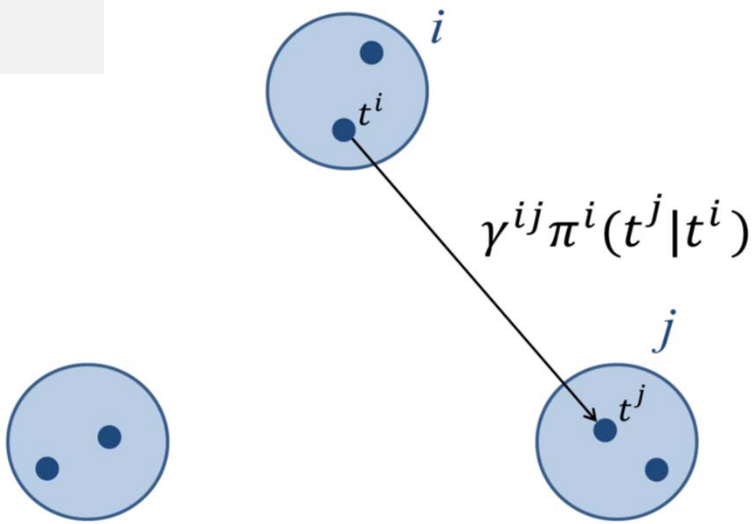

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Ballester, Calvó-Armengol, and Zenou (*Econometrica* 2006) “Who's Who in Networks”

Calvó-Armengol, de Marti, and Prat (*TE* 2015) “Communication and Influence”

Bergemann, Heumann, Morris (2017) "Information and Interaction“

Myatt and Wallace (2017), “Information Acquisition and Use by Networked Players”

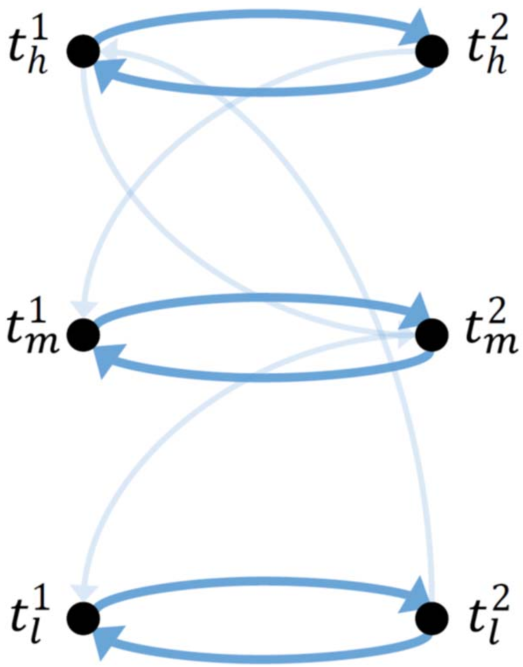
Tyranny of Least-Informed

Prop 3 Suppose $q^1 \leq 1-\delta$ at least δ-noisy

for all $i \neq 1$ $q^i \geq 1-\varepsilon$ at most ε-noisy

Then

$$\left| c(y;\vec{\pi}) - \mathbb{E}^{\rho^1}[y] \right| \leqslant K \cdot \frac{\varepsilon}{\delta}$$



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Ex 2 agents, incomplete info

$$-u^i = \beta(a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2$$

$\theta \in \{\theta_1, \dots, \theta_K\}$

$\rho^i \in \Delta(\Theta)$ i's prior

$t^i \in \{t_1^i, \dots, t_K^i\}$ matches θ w.p. q^i

Otherwise full support noise.

Tyranny of Least-Informed

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for all $i \neq 1$ $q^i \ge 1-\varepsilon$ at most ε-noisy

Then

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Proof Idea

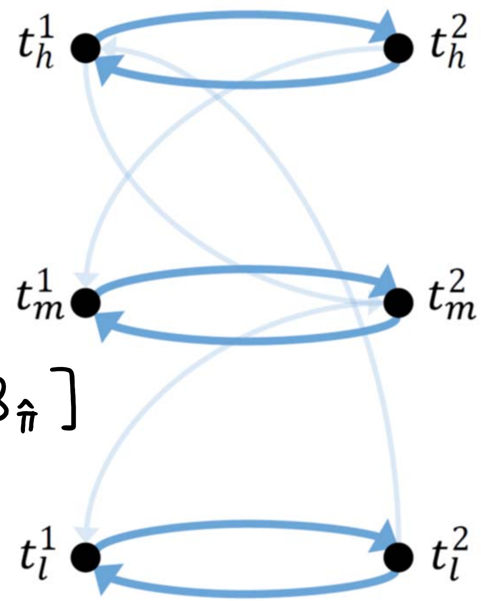
0. Define artificial $\hat{\pi}$:
   - each $i \neq 1$ knows θ
   - 1's info. unchanged

1. $c(y;\hat{\pi}) = \mathbb{E}^{\rho^1}[y]$

Reason: $\hat{\pi}$ satisfies CPs with 1's prior.

2. $p(B_{\vec{\pi}}) \approx p(B_{\hat{\pi}})$

Reason: if $\| B_{\vec{\pi}} - B_{\hat{\pi}} \| \times$ [max. mean 1st-passage time in $B_{\hat{\pi}}$] is small then $\approx$ holds.



Cho and Meyer (00) "Markov chain sensitivity measured by mean first passage times"

Conclusion

Interaction structure captures (interim) beliefs and network simultaneously: a method for studying how behavior depends on

(i) information (signals) | (ii) interpretation (priors) | (iii) coordination concerns (network)

General characterization of conventions in terms of eigenvector **centrality** in **interaction structure**. Reduction to a complete-information network game.

Illustrate with three applications.

Contagion of optimism – small local bias (in common direction) leads to extreme conventions.

Under **common prior over signals**, agents' prior expectations matter in proportion to their **centrality in the network** Γ only.

Under **common interpretation of signals** and precise private information, get **tyranny of the least-informed**.