DeGroot Updating Model

Reading: G& Sadler. Learning in Social Networks. *Handbook*.

BASIC SETUP

X - convex subset of a vector space

e.g. $X = [0,1]$ or $X = \Delta(\Omega)$ French (56), Harary (59), DeGroot (74, JASA)

$N = \{1, 2, \ldots, n\}$: agents

$x_i(t)$: estimate or "opinion" of i

updating rule : for $t \in \{1, 2, \ldots\}$

$$x_i(t) = \sum_j \underset{\uparrow \text{updating weights}}{W_{ij}} x_j(t-1).$$

Note $x_i(0)$ exogenous. W is $n \times n$ matrix with nonnegative entries; each row sums to 1. Simple, stationary, synchronous.

Simple example: $W_{ij} = 1/d_i$ for each i. Animations.

Discuss: interpretation of weights, arrow directions.

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MATRIX POWERS & CONNECTION w/ MARKOV

Updating rule in matrix form:

$$x(t) = \overset{\text{want to study}}{\overbrace{W^t}} x(0)$$

Interpretations

1. $$(W^t)_{ij} = \frac{\partial x_i(t)}{\partial x_j(0)}$$ effect on i's time-t estimate of j's initial estimate

2. $$(W^t)_{ij} = \sum_{s \in \mathcal{W}^t(i,j)} \omega(s)$$ sum, over all conduits of j's t-step influence on i, of the strength of that influence

walks of length t from i to j

Consider $s = (s(1), s(2), \ldots, s(L+1))$

$$\omega(s) := \prod_{\ell=1}^{L} W_{s(\ell)s(\ell+1)}$$

the weight of a walk is the product of all the edge weights.

$$(W^3)_{62} = W_{61} W_{13} W_{32} + W_{64} W_{45} W_{52}$$

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Markov iteration: $\pi(t) = \pi(0) W^t$

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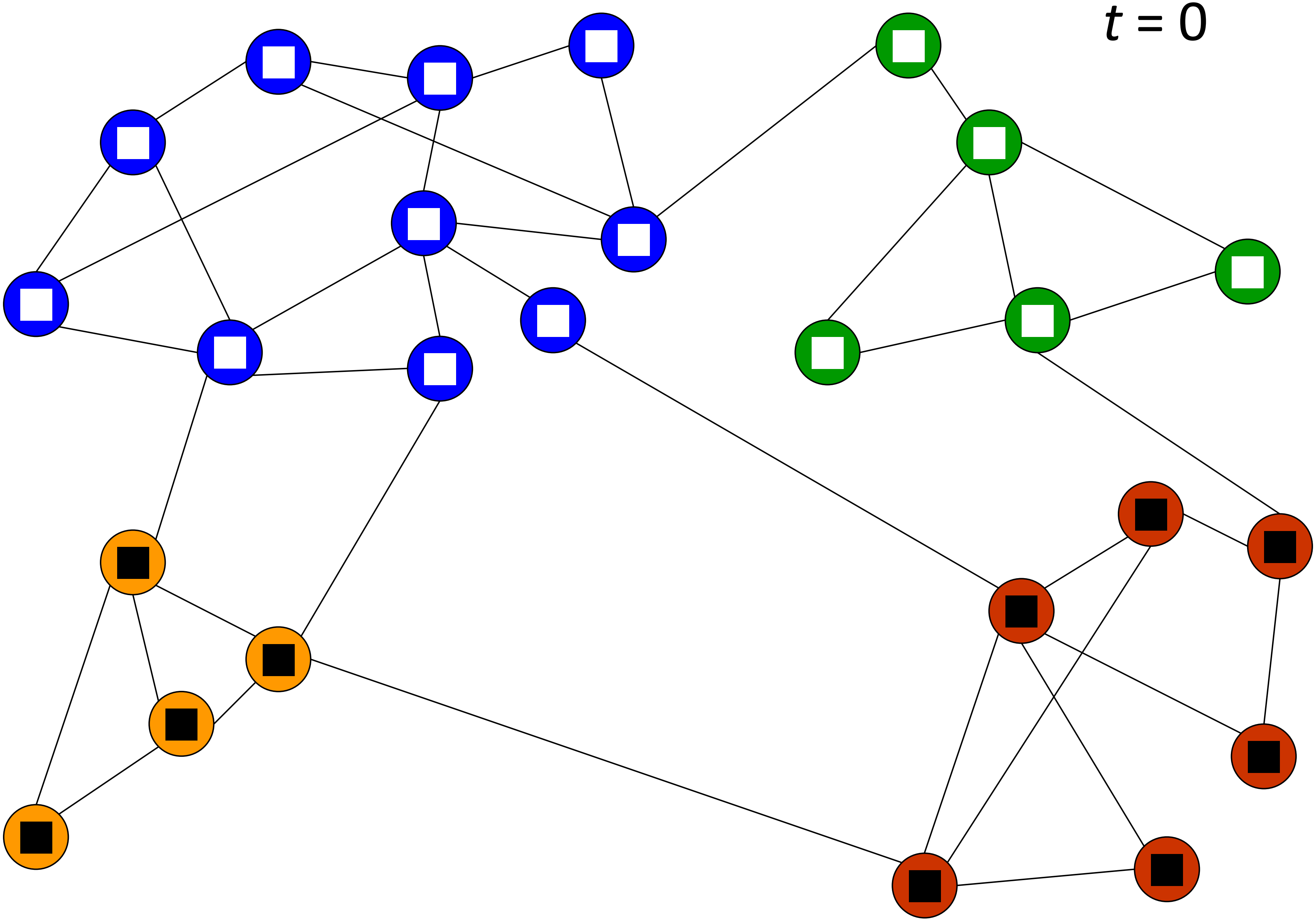
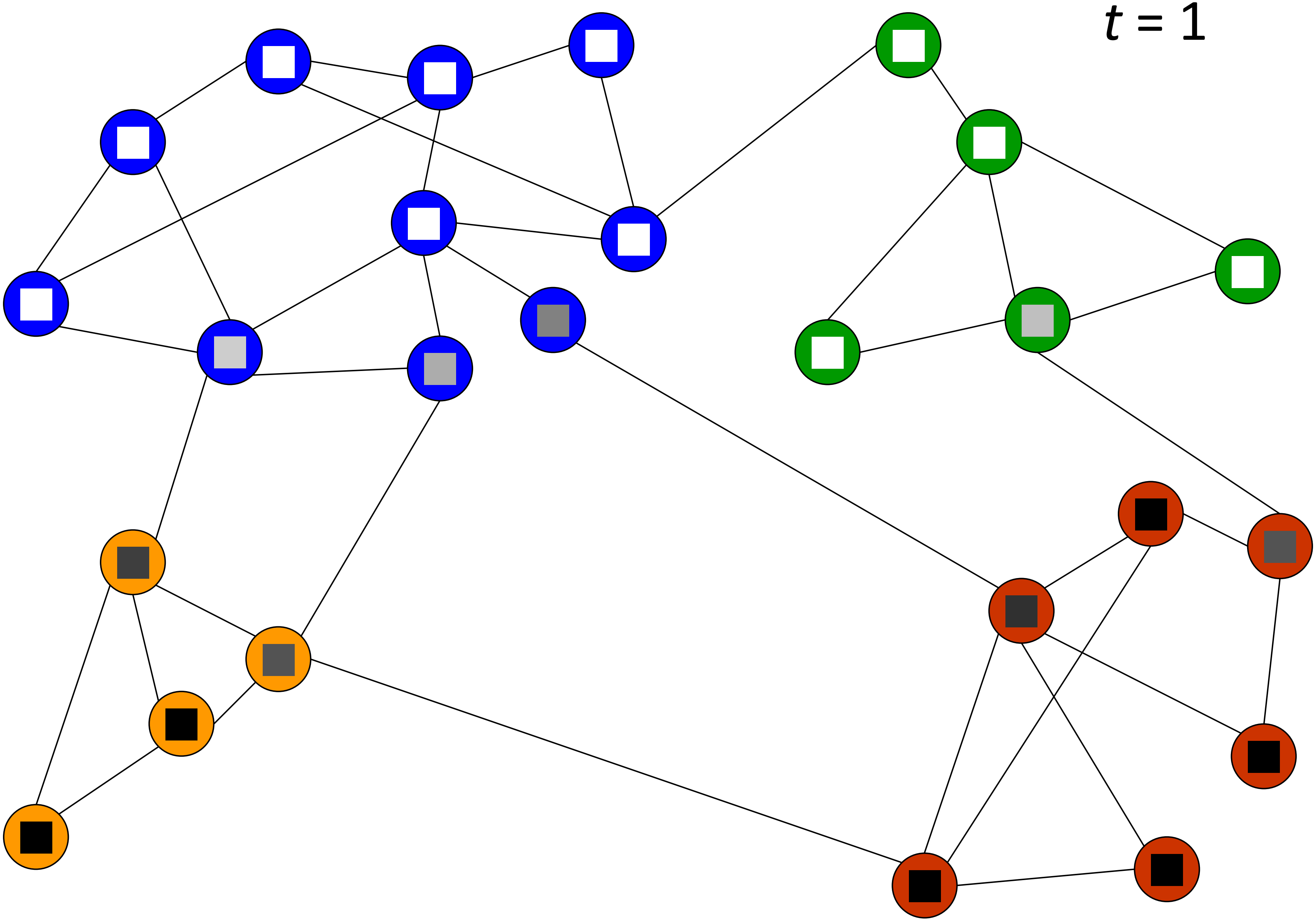
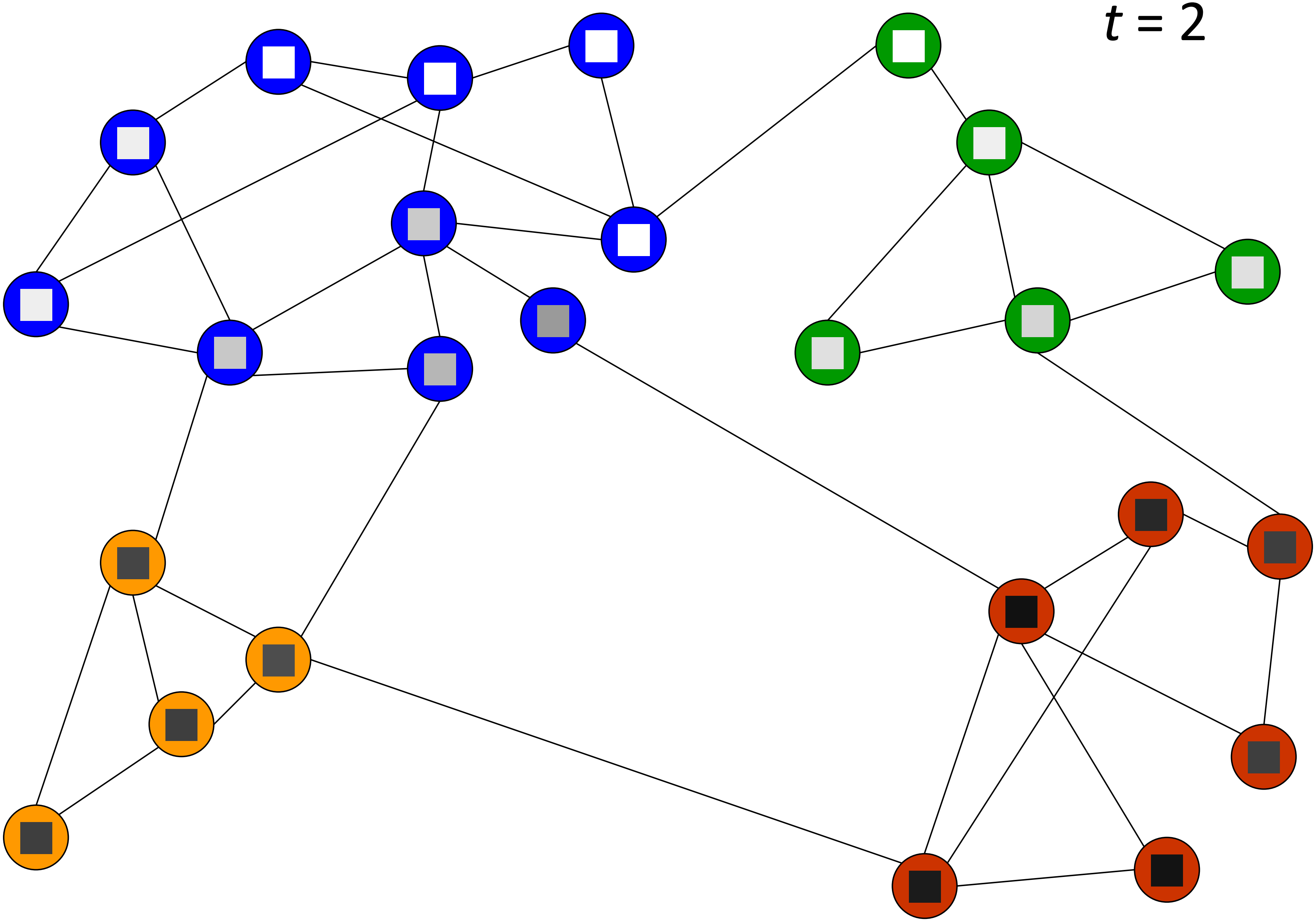
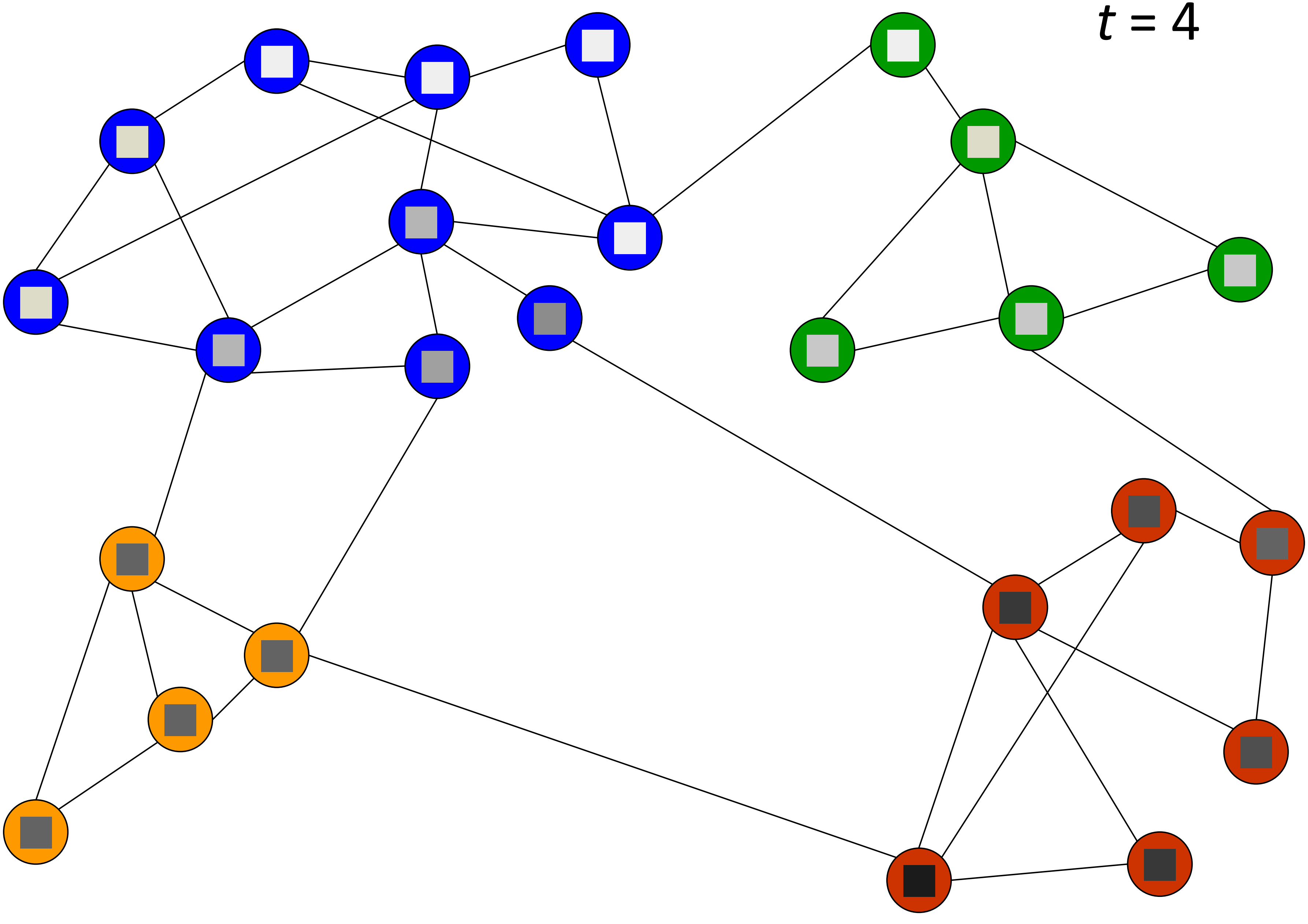
$t = 0$

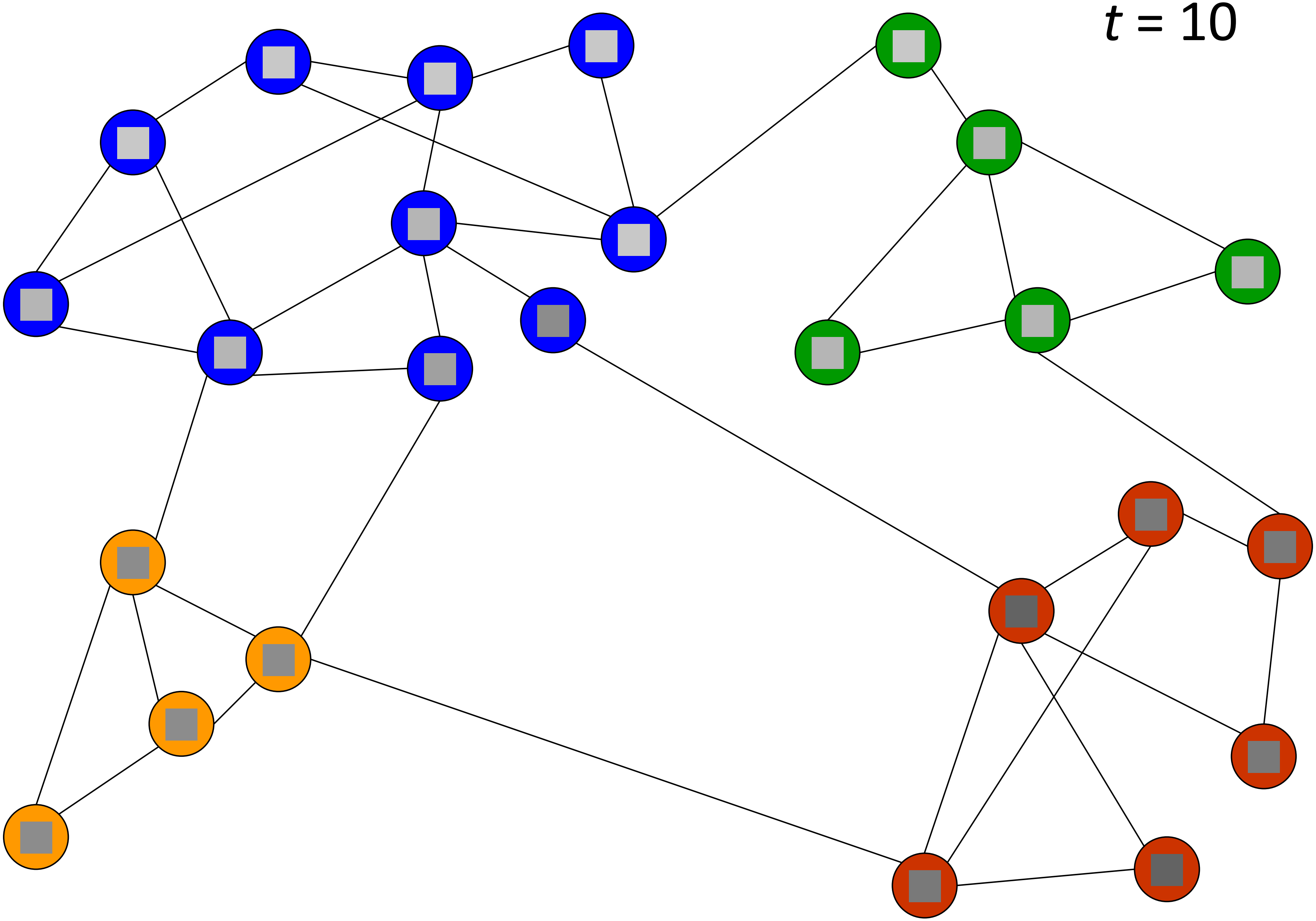


$t = 1$




$t = 2$


$t = 4$



t = 10

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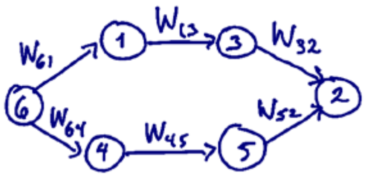

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Note $x_i(0)$ exogenous. W is $n \times n$ matrix with nonnegative entries; each row sums to 1. Simple, stationary, synchronous.

Simple example: $W_{ij} = 1/d_i$ for each i. Animations.

Discuss: interpretation of weights, arrow directions.

Foundations; variations: time-dependent weights $W(t)$; stochastic meetings; continuous time; persistent "own estimate"; discrete rule – later!

MATRIX POWERS & CONNECTION w/ MARKOV

Updating rule in matrix form:

$$x(t) = \overset{\text{want to study}}{W^t} x(0)$$

Interpretations

1. $(W^t)_{ij} = \dfrac{\partial x_i(t)}{\partial x_j(0)}$ effect on i's time-t estimate of j's initial estimate

2. $(W^t)_{ij} = \displaystyle\sum_{s \in W^t(i,j)} w(s)$ sum, over all conduits of j's t-step influence on i, of the strength of that influence

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Consider
$s = (s(1), s(2), \ldots, s(L+1))$

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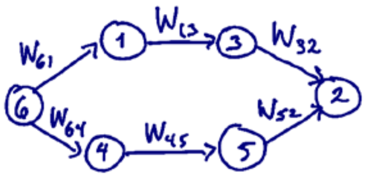
the weight of a walk is the product of all the edge weights.



$$(W^3)_{62} = W_{61}W_{13}W_{32} + W_{64}W_{45}W_{52}$$

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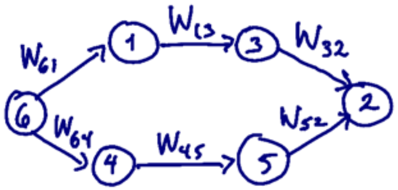

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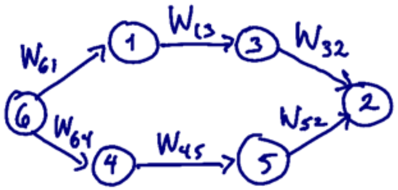

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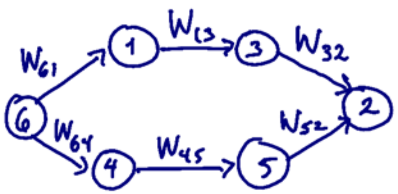

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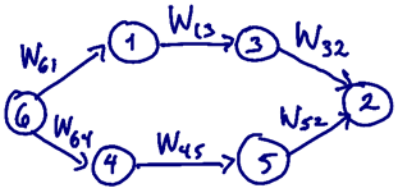

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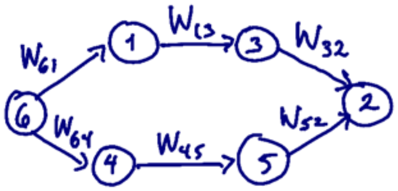

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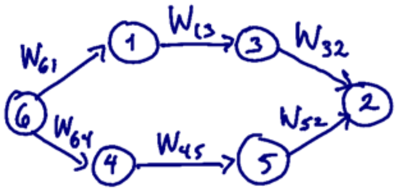

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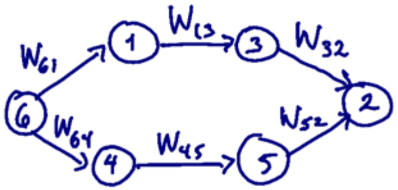

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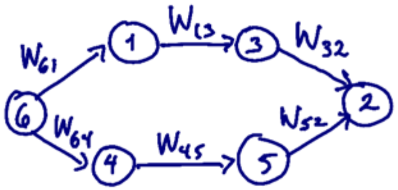

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$$x(\infty) = (1-\beta)\sum_{\ell=0}^{\infty}\beta^{\ell}W^{\ell}b$$

Friedkin + Johnsen (99)
G + Morris (WP 2016)

2. Quasi-Bayesian Learning / Correlation Neglect / Echo Chambers.

μ - true state of the world; i has diffuse normal prior, and signal $s_i = \mu + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2)$. Now $x_i(0)$ – i's period 0 estimate is s_i.

ε_i are independent, so

$$x_i(1) = \sum W_{ij}x_j(0)$$

captures precisions

DeGroot agents continue using this rule in later periods, too, neglecting that $x_i(1)$ are not conditionally independent given μ. (Enke 16)

3. Recently: Jadbabaie et al. (2012)

$$\mu_i(t) \in \Delta(\Omega) \xrightarrow[\text{Bayes}]{} \tilde{\mu}_i(t) \xrightarrow[\text{DeGroot}]{} \mu_i(t+1)$$

$s_i(t)$ – signals

$(\tilde{\mu}_j(t))$ – others' beliefs

consistent with some views of Savage, Harsanyi; compare w/ Aumann.

Recent axiomatization of DeGroot (in log odds ratios) by Jadbabaie, Molavi, and Tahbaz-Salehi (2016).

MATRIX POWERS & CONNECTION w/ MARKOV

Updating rule in matrix form:

$$x(t) = W^t x(0)$$

want to study

Interpretations

1. $(W^t)_{ij} = \dfrac{\partial x_i(t)}{\partial x_j(0)}$ effect on i's time-t estimate of j's initial estimate

2. $(W^t)_{ij} = \sum_{s \in W^t(i,j)} w(s)$ sum, over all conduits of j's t-step influence on i, of the strength of that influence

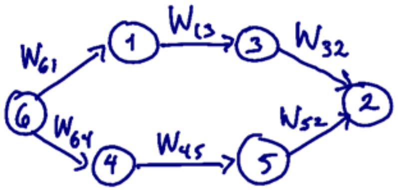
walks of length t from i to j

Consider

$s = (s(1), s(2), \ldots, s(L+1))$

$$w(s) := \prod_{\ell=1}^{L} W_{s(\ell)\,s(\ell+1)}$$

the weight of a walk is the product of all the edge weights.



$$(W^3)_{62} = W_{61}W_{13}W_{32} + W_{64}W_{45}W_{52}$$

Levin, Peres & Wilmer
Markov Chains
& Mixing Times

Markov iteration: $\pi(t) = \pi(0)W^t$

Let state j be labeled by y_j. Let $(Z_i(t))_{t=0}^{\infty}$ be Markov process started at i w/ transitions W.

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LONG-RUN LIMIT: CONSENSUS & CENTRALITY

Questions:

(i) $\lim_{t\to\infty} x_i(t)$ exists $\forall i$? (For all $x(0)$.)

(ii) If (i) holds, are limits same across i?

(iii) If (ii) holds, what is that consensus limit?

Preview: rich connection between outcomes and network structure will be evident in answer to (iii). Via eigenvector centrality!

(i,ii) Assume W strongly connected. Then answer to (i) and (ii) is "yes" for most W.
Necessary & sufficient conditions:
(a) $\exists \ell$ s.t. $W^\ell >> 0$: "W is primitive"
(b) gcd of lengths of simple cycles is 1. "W is aperiodic"

Idea of (a) ⇒ convergence: in ℓ steps of updating, the two most extreme agents place weight at least $\underline{W} > 0$ on each other. So

$$x_{max}(t+\ell) - x_{min}(t) \le (1-\underline{W})\left[x_{max}(t) - x_{min}(t)\right]$$

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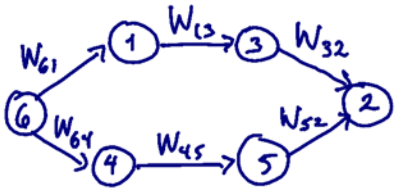

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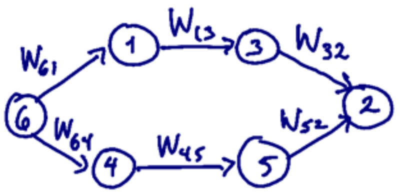

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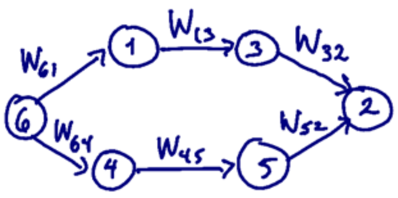

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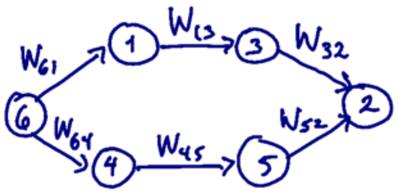

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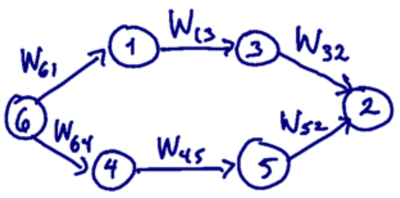

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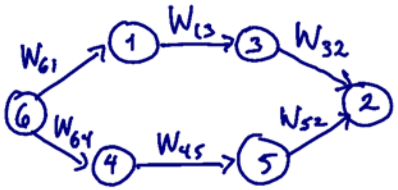

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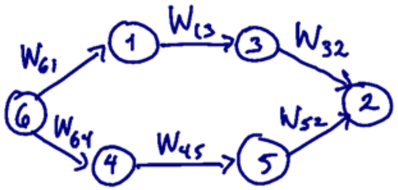

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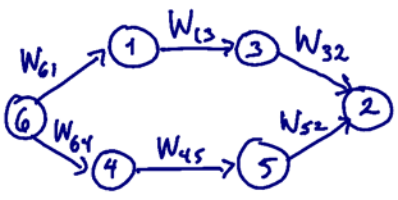

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$X(\infty) = W^\infty X(0)$. For (iii): what is W^∞?

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$$W^\infty = \begin{bmatrix} \text{---} & \pi^T & \text{---} \\ \text{---} & \pi^T & \text{---} \\ & \vdots & \\ \text{---} & \pi^T & \text{---} \end{bmatrix}$$

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So consensus no longer holds but analysis within each minimal closed component is same.

Reflections: Chicago; stubbornness. Discontinuity at ∞,

CORRECTNESS OF CONSENSUS. Recall DVZ (03) rationale for DeGroot. In same paper, they ask, is consensus equal to what a Bayesian would think with access to all signals? Generically no.

Now consider an asymptotic version of the question $\left(W(n)\right)_n \leftarrow \boxed{n \times n \text{ and convergent}}$ $x_i^{(n)}(0) = \mu + \varepsilon_i^{(n)}$

$\underline{\sigma}^2 \le Var[\varepsilon_i^{(n)}] \le \bar{\sigma}^2$. GJackson (10) Naive Learning in S. N. and the Wisdom of Crowds

Question: Does it hold that $\overbrace{x^{(n)}(\infty) \xrightarrow{p} \mu}^{\text{wisdom}}$?

Answer: Recall $x^{(n)}(\infty) = \sum_i \pi_i^{(n)} x_i^{(n)}(0)$.

When does this converge in probability to μ?

When $\boxed{\max_i \pi_i^{(n)} \xrightarrow{n\to\infty} 0}$ – a standard WLLN.

In GJ10 we give sufficient conditions in terms of the network.

In spirit: if there is a <u>prominent</u> group, then wisdom does not hold. If weights are "balanced" and society is reasonably "integrated" then wisdom does hold.

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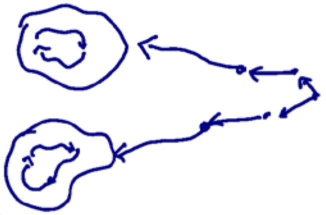

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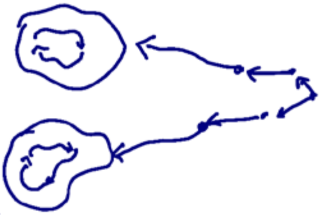

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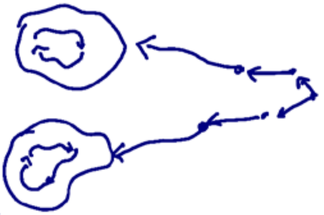

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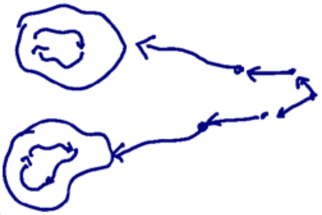

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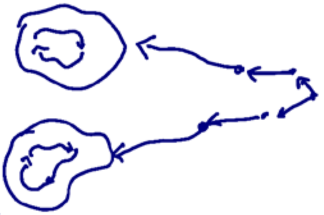

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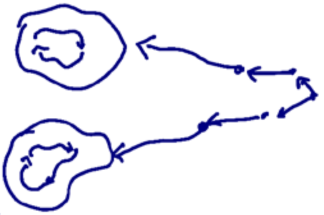

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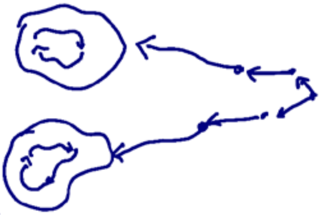

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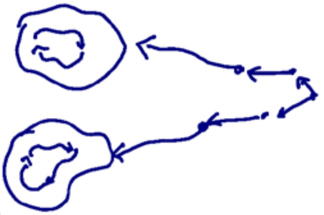

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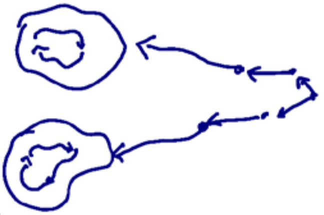

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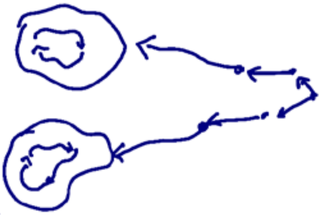

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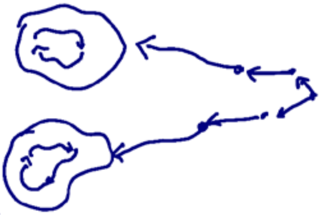

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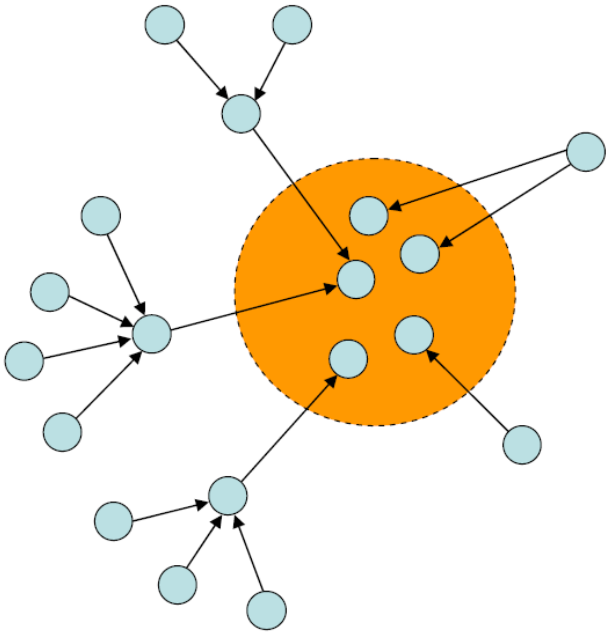

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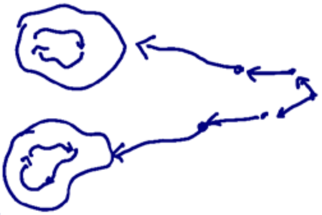

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To sum up: W strongly connected and primitive $\Rightarrow W^{\infty}$ exists.

$x(\infty) = W^{\infty} x(0)$. For (iii): what is W^{∞}?

<u>Fact</u>

$$W^{\infty} = \begin{bmatrix} \text{---} \; \pi^T \; \text{---} \\ \text{---} \; \pi^T \; \text{---} \\ \vdots \\ \text{---} \; \pi^T \; \text{---} \end{bmatrix}$$

where π is normalized left Perron vector of W. a.k.a. the stationary distribution of W if we view W as a Markov matrix.

So $\forall i$, $x_i(\infty) = \sum_i \pi_i x_i(0)$, answering (iii)

Closer look at π: $\pi_i = \sum_j W_{ji} \pi_j$

Why is π like that? $W^{\infty} W = W^{\infty}$

$\pi^T W = \pi^T$ a typical row of

Clearly $\pi \gneq 0$.

By strong connectedness of W, $\pi \gg 0$. (Exercise.)

By Perr.-Fr., π is the <u>unique</u> nonnegative nonzero $\hat{\pi}$ satisfying $\hat{\pi}^T W = \hat{\pi}^T$.

Intuitive: i's long-run influence is a weighted sum of influences of those who pay attention to i, weighted by the attention.

So, in answer to our questions: (i) $\forall i$, $x_i(t) \to$ a limit; (ii) they all converge to the <u>same</u> limit, $x(\infty)$; (iii) this limit is $\sum_i \pi_i x_i(0)$, where π_i is i's eigenvector centrality in W.

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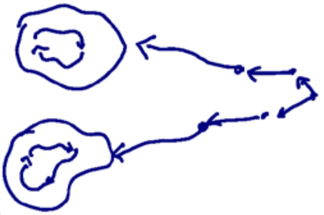

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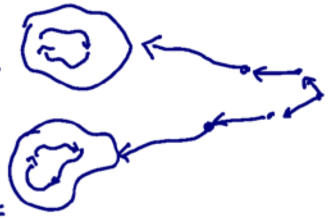

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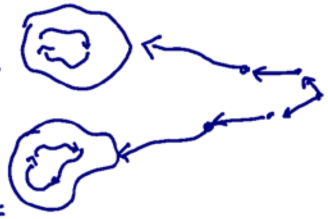

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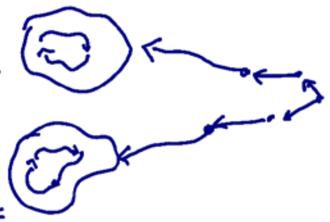

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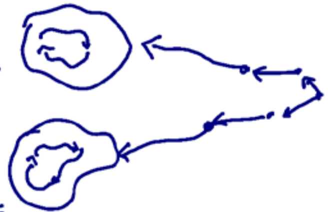

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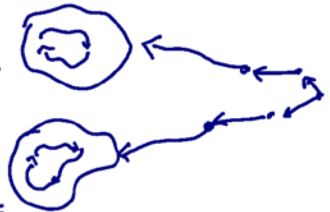

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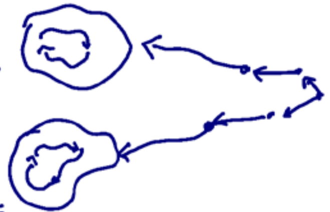

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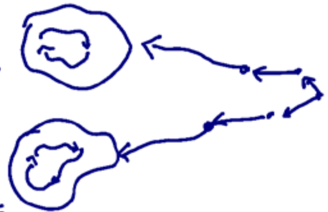

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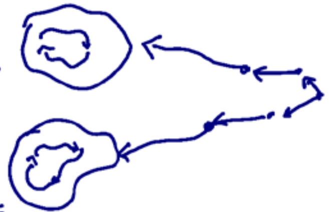

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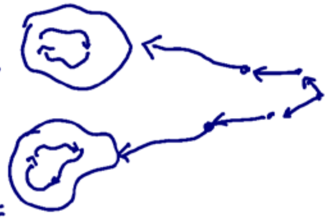

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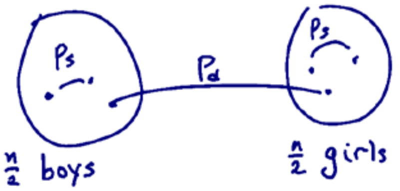

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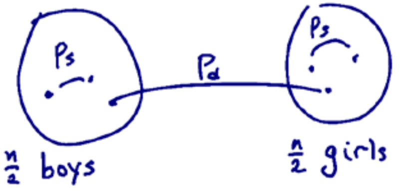

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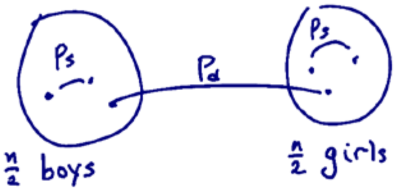

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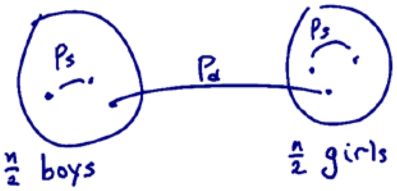

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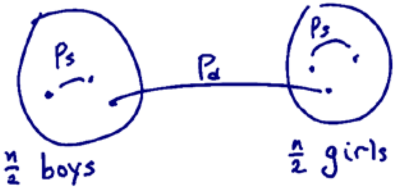

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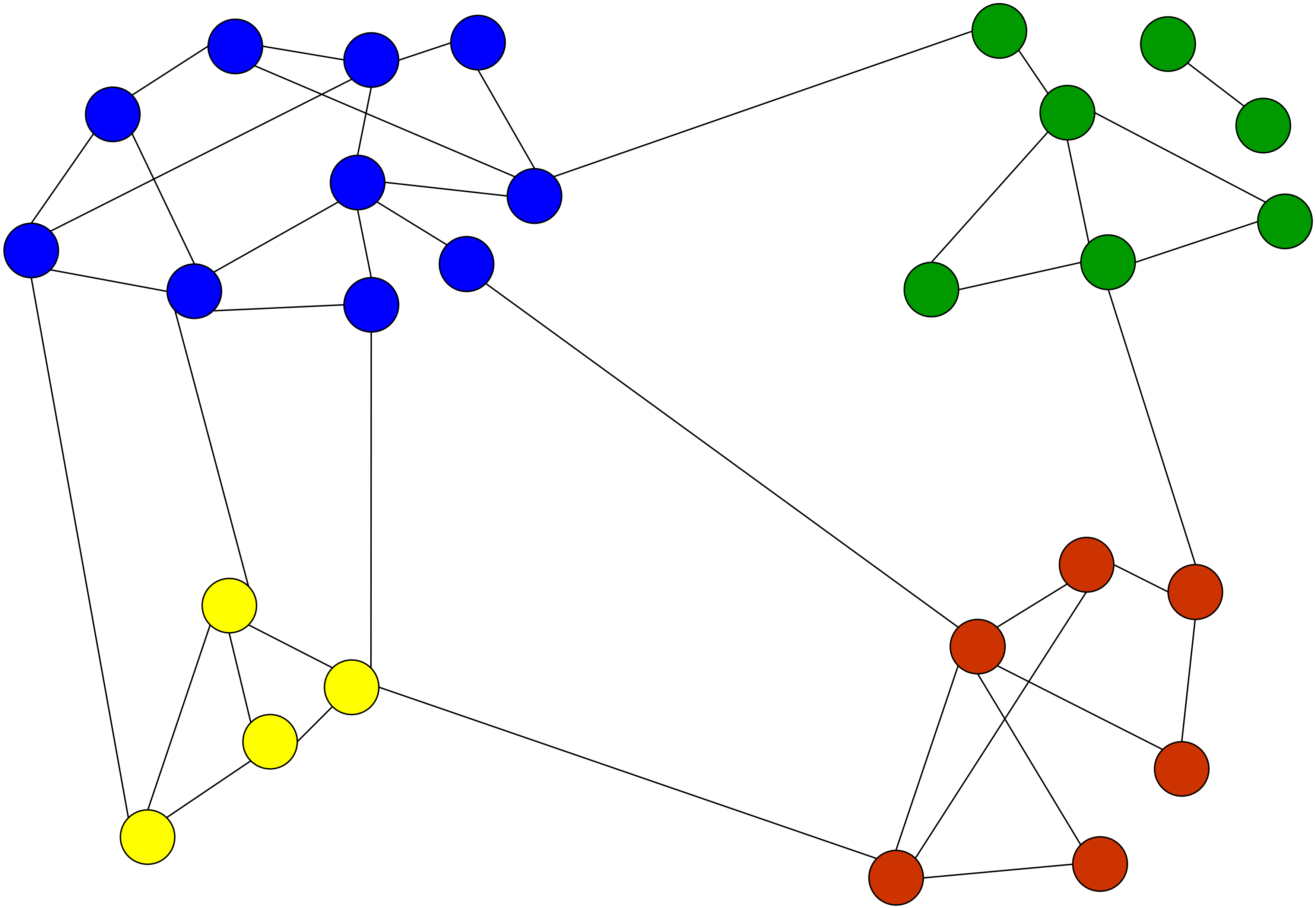

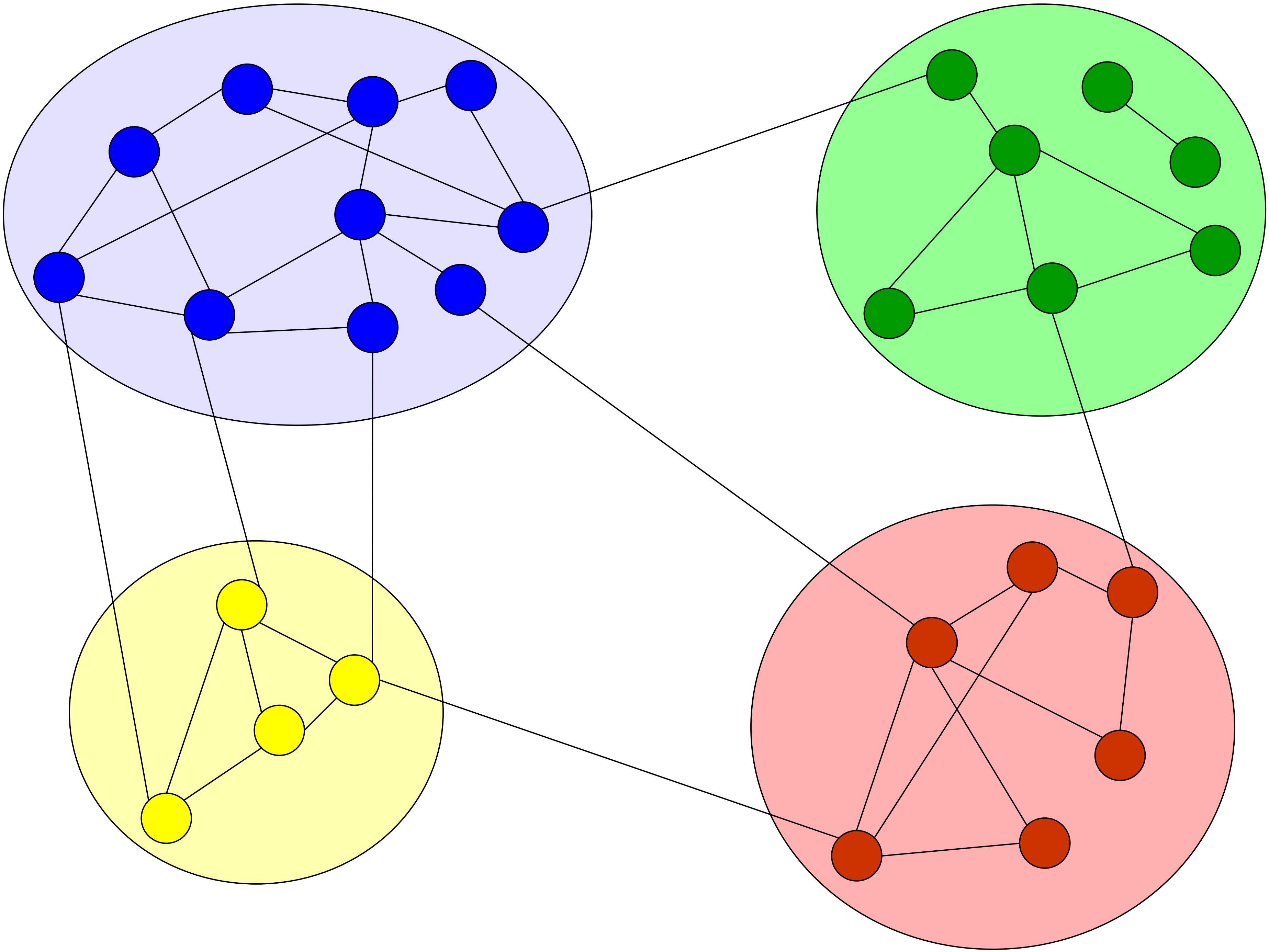

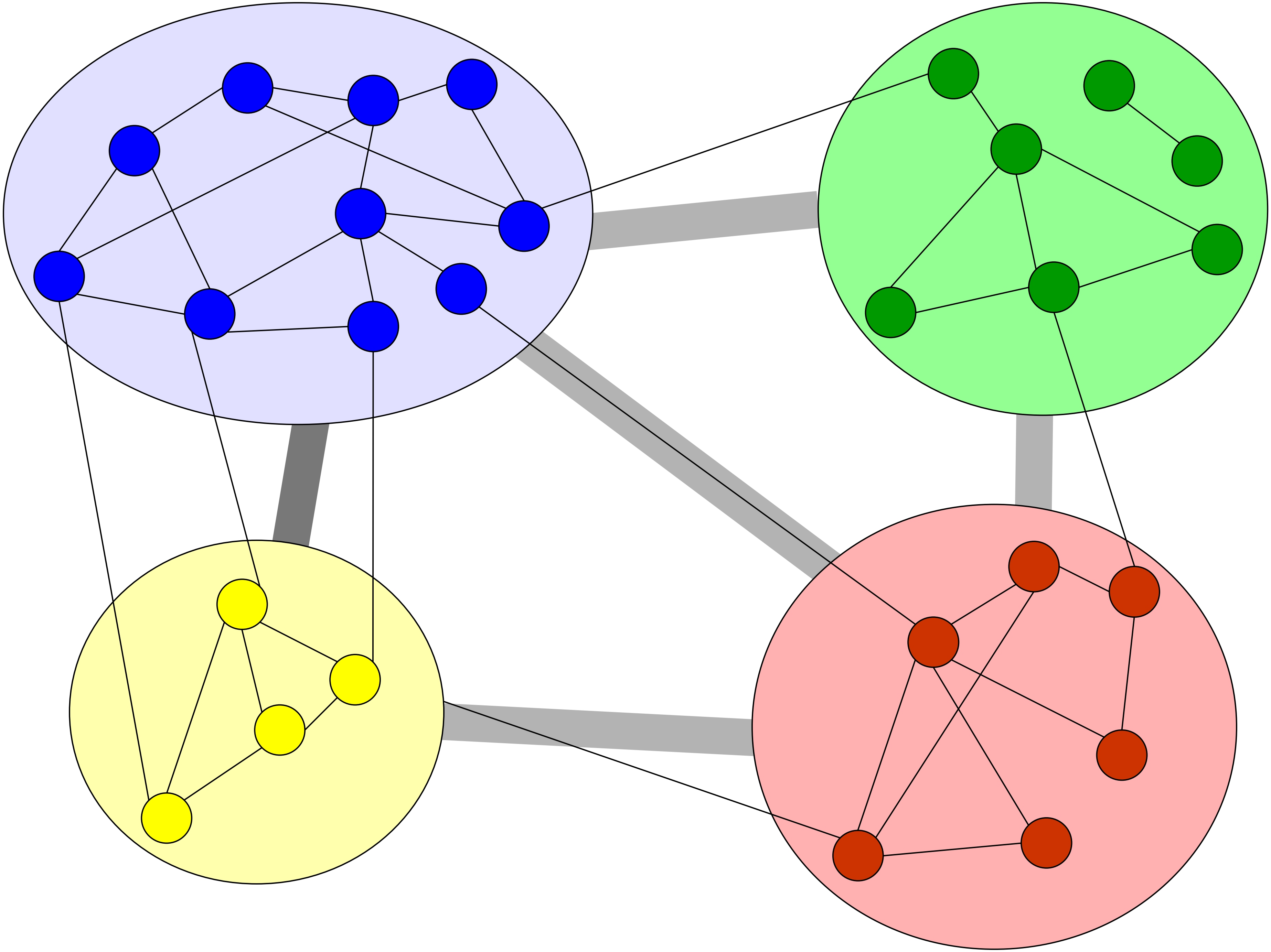

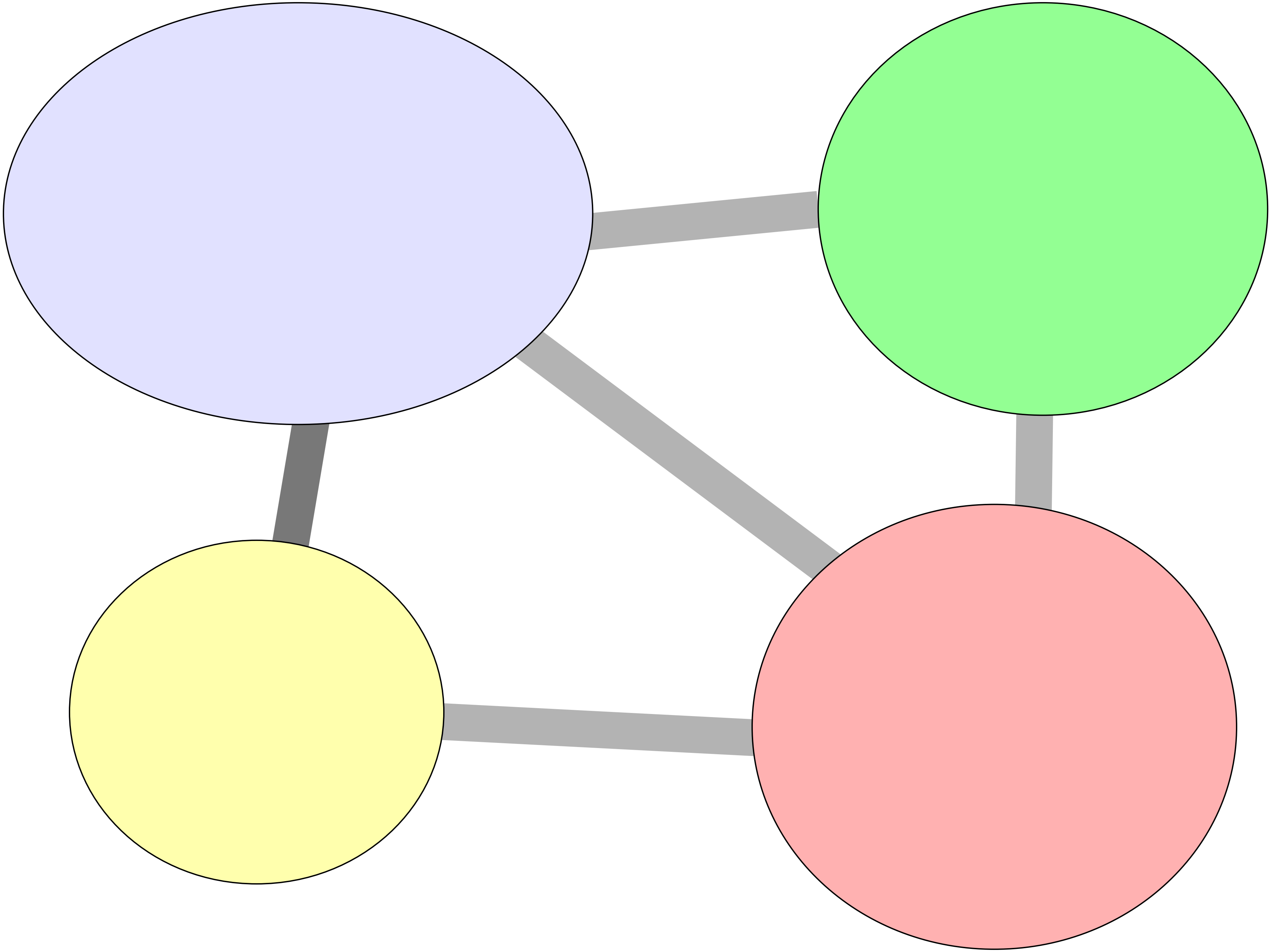

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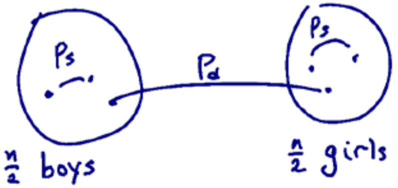

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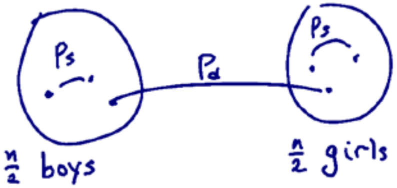

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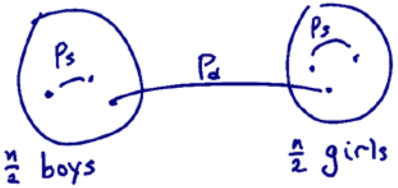

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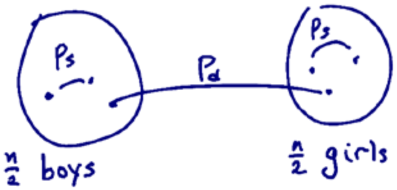

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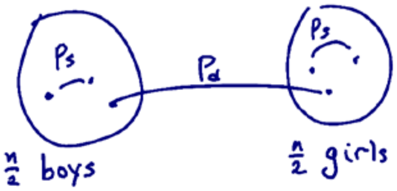

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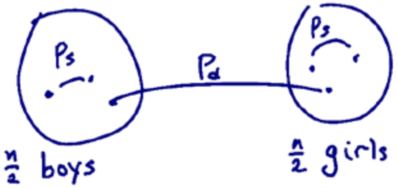

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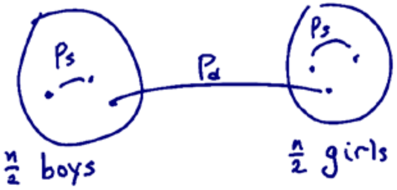

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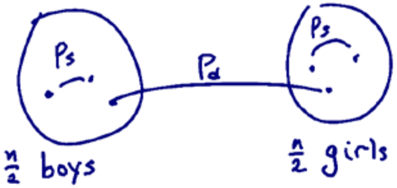

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Moreover, get consensus under very similar conditions to deterministic case, by similar arguments. (Tahbaz-Salehi & Jadbabaie 2008)

2. Continuous time process with Poisson clocks determining activity.

3. Stubborn agents, random meetings
   Acemoglu, Como, Fagnani, Ozdaglar (13)
   Acemoglu, Ozdaglar, ParandehGheibi (10)

4. Contrast with fully rational
   - learn all signals in $O(n^2)$ periods
   - not truly intractable - just "wrong" Mossel, Olsman, Tamuz
   - G-Polemarchakis process in Aumann world ...