Naive Learning in Social Networks and the Wisdom of Crowds

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  - Many interesting networks have poor learning; many also have good learning.

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- The initial beliefs $b_i(0)$ are independent random draws with mean θ and all lie in the same compact set $[-K, K]$.

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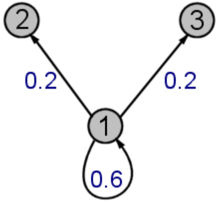

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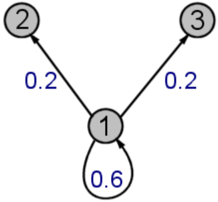

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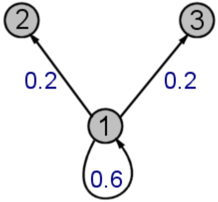

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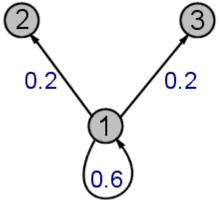

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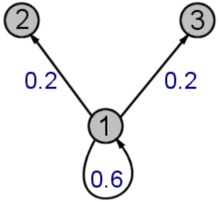

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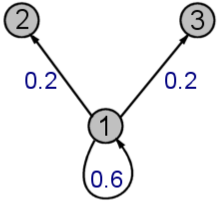

$$b_i(t+1) = \sum_{j \in A} T_{ij} b_j(t)$$

where

$$\sum_{j \in A} T_{ij} = 1.$$

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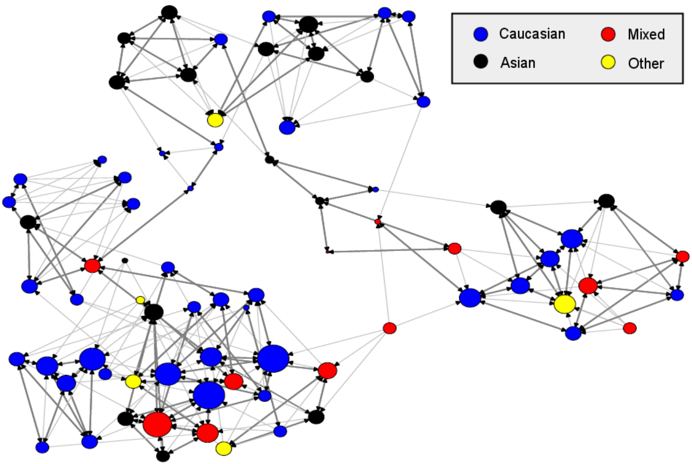
Also, $\sum_{j \in A} T_{ij} = 1 \;\Rightarrow\;$ each row of $\mathbf{T}$ sums to 1.

The Social Network

The matrix **T** naturally corresponds to a social network. The entry T_{ij} describes the "trust" or "weight" that agent i places on the beliefs of agent j in forming his next-period beliefs.

Friendships at Westridge School



Jacob K. Goeree, Maggie McConnell, Tiffany Mitchell, Tracey Tromp, and Leeat Yariv, A simple $1/d$ law of giving, mimeo., Caltech, 2006.

Convergence

Under some fairly mild conditions, the belief of each individual i eventually settles down to some limit

$$b_i(\infty) = \lim_{t\to\infty} b_i(t).$$

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- Each society n has an associated vector of beliefs evolving over time: $\mathbf{b}^{(n)}(t)$.
- Assume beliefs in every society converge; let the vector of limiting beliefs in society n be $\mathbf{b}^{(n)}(\infty)$.

Definition of Wisdom

Wisdom means that, as society grows large, limiting beliefs converge to the truth.

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Definition

The sequence $(\mathbf{T}^{(n)})$ is *wise* if

$$\operatorname*{plim}_{n\to\infty} \max_{i\in A_n} |b_i^{(n)}(\infty) - \theta| = 0.$$

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- Write

$$T_{i,B}(p) = \sum_{j \in B} T_{ij}(p).$$

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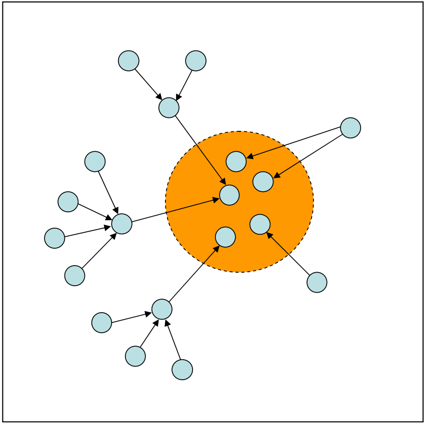
Call $\pi_B(\mathbf{T}; p) = \min_{i \notin B} T_{i,B}(p)$ the *p-step prominence* of B relative to $\mathbf{T}$.

Example of a Prominent Group



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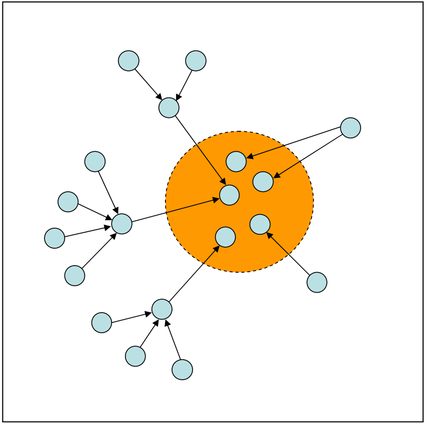

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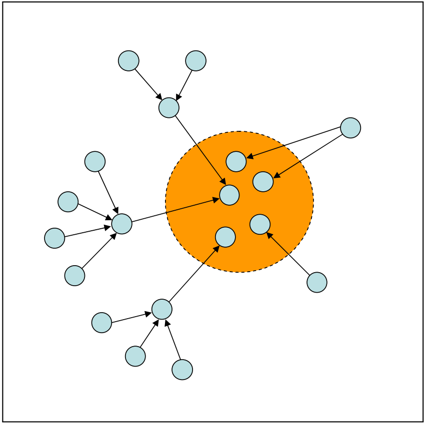

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Note that the rest of **T** can be completed arbitrarily.

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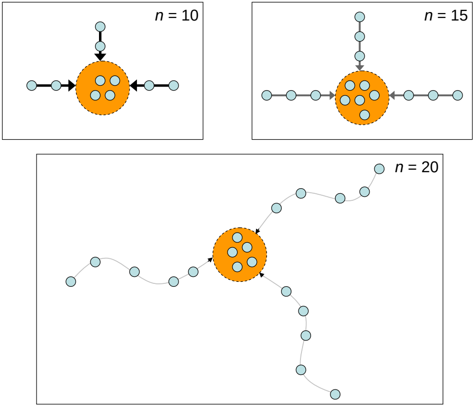
- Intuitively: (B_n) is uniformly prominent with respect to $(\mathbf{T}^{(n)})$ means:
  - Each B_n is a prominent group with respect to $\mathbf{T}^{(n)}$.
  - The prominence does not decay to 0.

Prominent Families: What We Are Ruling Out



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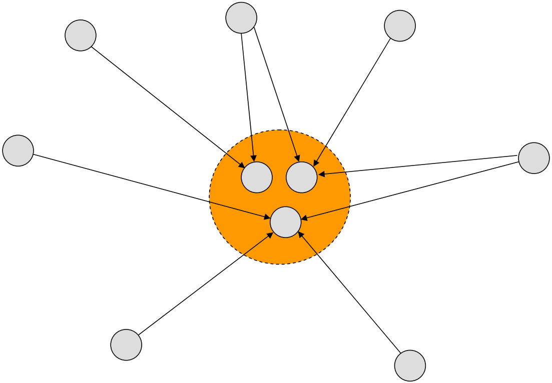
The family (B_n) is *uniformly prominent* relative to $(\mathbf{T}^{(n)})$ if there exists a constant $\mu > 0$ so that for each n, there is a p so that $\pi_{B_n}(\mathbf{T}; p) \geq \mu$.

Small Prominent Families Prevent Wisdom

Proposition

If there is a finite, uniformly prominent family with respect to $(\mathbf{T}^{(n)})$, then the sequence is not wise.

Intuition



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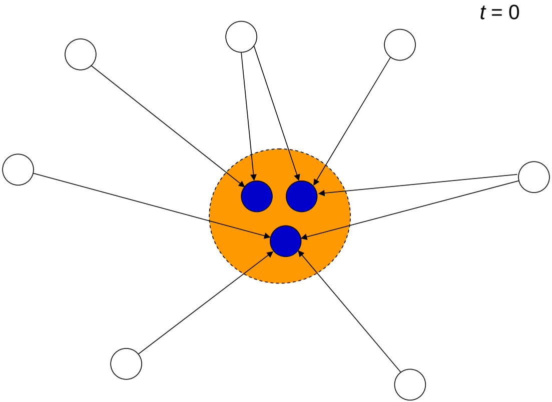

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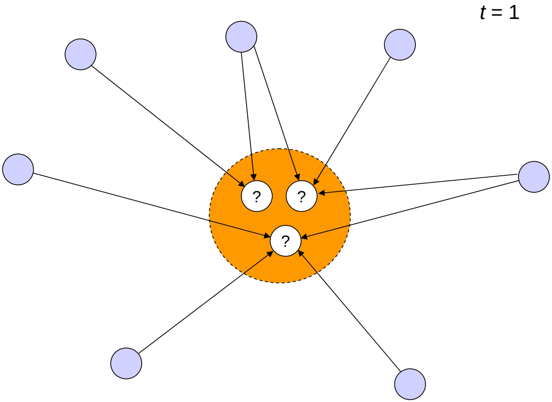

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Theorem

If $(\mathbf{T}^{(n)})$ satisfies balance and minimum out-dispersion, then it is wise.

Main Conclusions

- Small prominent groups (media, pundits) are bad for information aggregation when agents are naive.
- Balance and dispersion conditions can guarantee wisdom.

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- Nonhomogeneous updating (updating matrix changes).