# Targeting interventions in networks

Andrea Galeotti (LBS), Benjamin Golub (Harvard)
and Sanjeev Goyal (Cambridge)

2019

# Question

Simultaneous move, $n$ agents. Agent $i$ chooses investment $a_i$ to maximize

$$W_i(a_i, \boldsymbol{a}_{-i}; b_i) = a_i \cdot R(b_i, \boldsymbol{a}_{-i}) - \frac{1}{2}a_i^2$$

where

$$R(b_i, \boldsymbol{a}_{-i}) = \underbrace{b_i}_{\substack{\text{basic/standalone} \\ \text{marginal return}}} + \underbrace{\beta}_{\substack{\beta > 0 \text{ strat. comp.} \\ \beta < 0 \text{ strat. subst.}}} \sum_j \overbrace{g_{ij}}^{\text{network}} a_j$$

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*Targeted interventions* modify, at a cost, incentives of some. *Goal:* max. (e.g.) **utilitarian welfare**. *Question:* **Whom to target and in what proportions?**

# Contribution

A. Study optimal targeting under strategic complements or substitutes, positive or negative externalities, various objective functions.

   Dependence on (i) *network structure*, on (ii) *nature of strategic interaction*, (iii) *intervention size*, etc.

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B. Look at game *in a different basis*, where strategic amplification has a simple structure: "principal component approach."

C. Optimal targeting can be expressed simply in this new basis: interventions with highest leverage in a given setting are proportional to certain **principal components** of the matrix of interaction.

# Targeting problem

$$W_i(a_i, \boldsymbol{a}_{-i}; b_i) = a_i \cdot \left[ b_i + \beta \sum_j g_{ij} a_j \right] - \frac{1}{2} a_i^2$$

$$\begin{aligned} \text{choose } \Delta\boldsymbol{b} \text{ to maximize} \qquad & \sum_i W_i(\boldsymbol{a}^*; \boldsymbol{b}) \text{ s.t. } \boldsymbol{a}^* \text{ being a Nash equilibrium,} \\ & \boldsymbol{b} = \hat{\boldsymbol{b}} + \Delta\boldsymbol{b} \\ \text{and} \qquad & K(\Delta\boldsymbol{b}) \leq C, \end{aligned}$$

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Assumption of unique equilibrium: $r(\beta\boldsymbol{g}) < 1$.
Assumption on $\boldsymbol{g}$: symmetric and nonnegative
(simplifies exposition a lot, but not essential).

$$\boldsymbol{g} = \overbrace{\underbrace{\begin{bmatrix} \vdots & & \vdots \\ \boldsymbol{u}^1 & \dots & \boldsymbol{u}^n \\ \vdots & & \vdots \end{bmatrix}}_{\boldsymbol{U}\text{: eigenvectors}} \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{\boldsymbol{\Lambda}\text{: eigenvalues}} \underbrace{\begin{bmatrix} \dots & \left(\boldsymbol{u}^1\right)^T & \dots \\ & \vdots & \\ \dots & \left(\boldsymbol{u}^n\right)^T & \dots \end{bmatrix}}_{\boldsymbol{U}^T\text{: eigenvectors}}}^{\text{Eigenvalue decomposition}}$$

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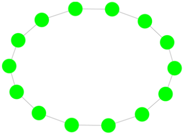
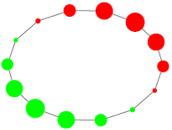
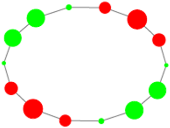


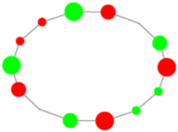
$\ell = 1$



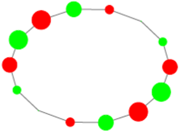
$\ell = 2$



$\ell = 4$



$\ell = 10$



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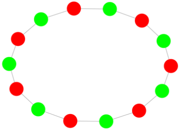

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# Targeting theorem for large budgets

If $C$ is large enough and ...

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2. $\ldots \beta < 0$, then $(\Delta \boldsymbol{b})^* \approx c\boldsymbol{u}^n$.

   If the budget is large and there are **strategic substitutes**, target according to the last eigenvector.

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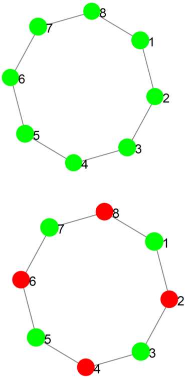
negative change ($\Delta b_i$)

*Size of circle: magnitude of change*




Recall: with strategic…

**complements**: target in proportion to $\boldsymbol{u}^1$ (*fact*: $u_i^1$ = eigenvector centrality); interventions focus on most **global** network response.

**substitutes**: divide communities, opposite treatment of neighbors; interventions focus on most **local** network structure.

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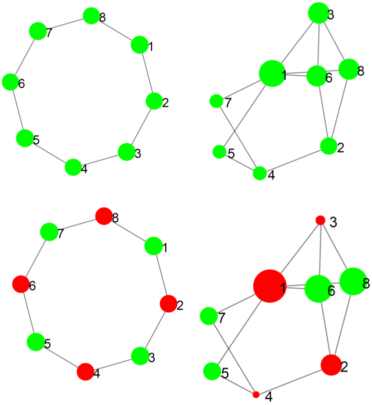


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# Literature

Social spillovers in important behaviors/outcomes: identification and evidence:

- Bramoullé, Djebbari, and Fortin (09); Calvò-Armengol, Patacchini, and Zenou (09); Bhuller, Dahl, Løken, Mogstad (18).

Network games:

- Ballester, Calvò-Armengol, and Zenou (06); Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (10); Bramoullé, Kranton, and d'Amours (14); Bimpikis, Ehsani, and İlkılıç (forthcoming)

Interventions and targeting in networks:

- Kempe, Kleinberg, and Tardos (03); Borgatti (06); Valente (12); Bloch (15); Belhaj and Deroian (17); Demange (17); Fainmesser and Galeotti (17)

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  - Components with **high** eigenvalues are more important in problems with strategic **complements**.
  - Components with **low** eigenvalues are more important in problems with strategic **substitutes**.

# Cosine Similarity

Definition:

- Cosine similarity of vectors $\boldsymbol{z}, \boldsymbol{y} \neq \mathbf{0}$ is $\rho(\boldsymbol{z}, \boldsymbol{y}) = \frac{\boldsymbol{z} \cdot \boldsymbol{y}}{\|\boldsymbol{z}\| \|\boldsymbol{y}\|}$.
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**Theorem 1: Characterization of Optimal Interventions**
At the optimal intervention, the similarity between $(\Delta \boldsymbol{b})^*$ and principal component $\boldsymbol{u}^\ell(\boldsymbol{g})$ satisfies the following proportionality:

$$\rho((\Delta \boldsymbol{b})^*, \boldsymbol{u}^\ell) \quad \propto \quad \rho(\hat{\boldsymbol{b}}, \boldsymbol{u}^\ell) \cdot r_\ell \tag{1}$$

where $r_\ell$ depends only on $\beta\lambda_\ell$ and the magnitude of the intervention (through shadow cost of the budget constraint).

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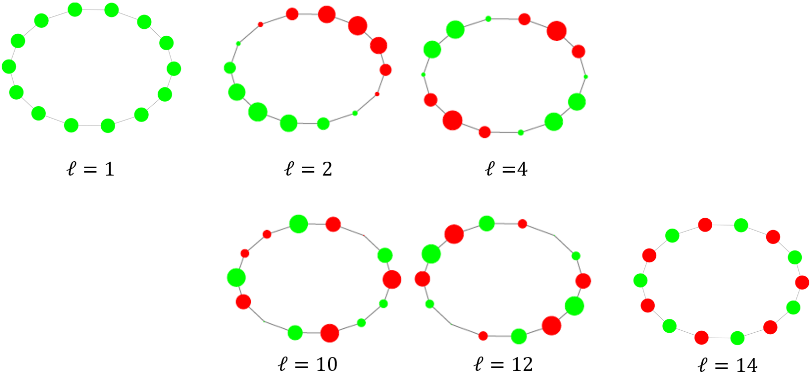
**Corollary 1: Monotonicity.** At the optimal intervention

1. If the game exhibits strategic complements the optimal intervention focuses more on the "higher" principal components, $r_1 > r_2 > \cdots > r_n > 0$
2. If the game exhibits strategic substitutes the optimal intervention focuses more on the "lower" principal components, $0 < r_1 < r_2 < \cdots < r_n$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The bound becomes easier to satisfy as the gap between the extreme and next eigenvalue (top or bottom spectral gap) grows.

# Large and small spectral gap (top)

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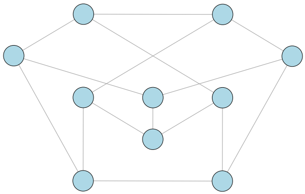

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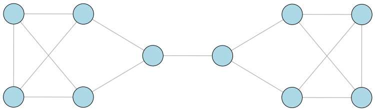

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- New network statistics matter: In strategic substitutes problems, focus on node statistics which reflect approximate local bipartitions.
- Extensions:
  - Nonsymmetric $\boldsymbol{G}$ (use singular value decomposition).
  - More general functional forms: e.g., small budget analysis.
  - More general externalities.
  - Incomplete information about $\boldsymbol{b}$.
  - Monetary incentives.
  - ...

# Proof idea

$$\text{recall } W_i(a_i, \boldsymbol{a}_{-i}; b_i) = a_i \cdot \left[ b_i + \beta \sum_j g_{ij} a_j \right] - \frac{1}{2} a_i^2$$

**New basis**:

$(\boldsymbol{I} - \beta \boldsymbol{g})\boldsymbol{a} = \boldsymbol{b} \underbrace{\text{iff}}_{\underline{\boldsymbol{z}} = \boldsymbol{U}^T \boldsymbol{z}} (\boldsymbol{I} - \beta \boldsymbol{\Lambda})\underline{\boldsymbol{a}} = \underline{\boldsymbol{b}}$

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**Objective:** In equilibrium, $W =$
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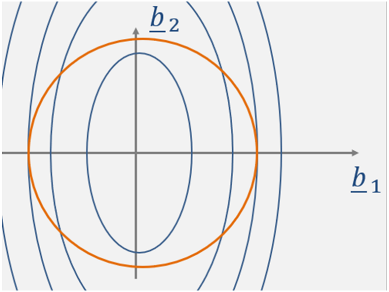


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**Constraint:** $\|\boldsymbol{b} - \hat{\boldsymbol{b}}\|_2^2 = \|\underline{\boldsymbol{b}} - \hat{\underline{\boldsymbol{b}}}\|_2^2 \leq C$

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$$\text{bound} = \frac{2}{\epsilon}\left(\frac{\alpha_2}{\alpha_1 - \alpha_2}\right)^2 \qquad \text{where } \alpha_\ell = (1 - \beta\lambda_\ell)^{-2}$$

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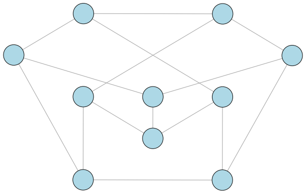

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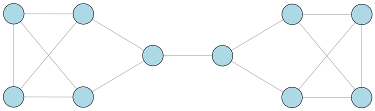

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# Generalizations and extensions

- We studied a game with strategic complements + positive externalities; strategic substitutes + negative externalities.

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- Providing incentives with money:
  - population version of model. If you want to effect a small change, you have to pay a small number of people a small amount of money each (since they are marginal);
  - paper on tax/subsidy interventions (Galeotti, Golub, Goyal, Talamàs, and Tamuz 19)

# Generalizations and extensions

- We studied a game with strategic complements + positive externalities; strategic substitutes + negative externalities.
- In paper: a framework flexible enough to handle any combination: e.g., a public goods game with strategic substitutes and positive externalities; also nest beauty contest games, etc. Principal component approach is portable.
- Providing incentives with money:
  - population version of model. If you want to effect a small change, you have to pay a small number of people a small amount of money each (since they are marginal);
  - paper on tax/subsidy interventions (Galeotti, Golub, Goyal, Talamàs, and Tamuz 19)
- Incomplete information: control $\Delta \boldsymbol{b}$ without knowing $\hat{\boldsymbol{b}}$.

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- Expected payoff to group $i$ is

$$U_i(\boldsymbol{a}^*, \boldsymbol{b}) = \int_0^{a_i^*} \left( \beta \sum_j g_{ij} a_j^* + b_i - \tau_i \right) d\tau_i = \int_0^{a_i^*} \left( a_i^* - \tau_i \right) d\tau_i = \frac{1}{2} {a_i^*}^2.$$

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- **Key observation:** Cost is quadratic in the size of the intervention: i.e., what mass of types have incentives changed.