

This problem set is worth 100 points. You should choose problems adding up to at least 100 points (write at the top of your problem set which ones you have chosen) and complete them.

Reading: [EK] 16, Osborne and Rubinstein 5.1–5.3

1. (25 points) EK 16.8 Exercise 3
2. (25 points) EK 16.8 Exercise 4
3. (25 points) EK 16.8 Exercise 5
4. (25 points) Osborne and Rubinstein Exercise 69.2
5. (40 points) In the notation of Osborne and Rubinstein Chapter 5, let  $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ , and suppose players 1 and 2 have information partitions

$$\mathcal{P}_1 = \{\{1, 2, 3, 4\}, \{5, 6\}, \{7\}, \{8\}, \{9, 10\}\}$$

$$\mathcal{P}_2 = \{\{1, 2\}, \{3, 4, 5\}, \{6\}, \{7, 8\}, \{9, 10\}\}$$

You can imagine rolling a 10-sided die, and telling player 1 which element of  $\mathcal{P}_1$  the result falls in, while telling player 2 which element of  $\mathcal{P}_2$  the result falls in.

An *event* is simply a subset of  $\Omega$ : a set of outcomes.

- a. (10 points) Describe the information partition that corresponds to knowing everything that player 1 knows and everything that player 2 knows – i.e., pooling their information.
- b. (5 points) When  $\omega = 3$  is the outcome of the die, what are the outcomes that player 1 considers possible? Call this set  $P_1(3)$ . What are the outcomes that player 2 considers possible? Call this set  $P_2(3)$ .
- c. (5 points) For any event  $F$ , define  $P_i(F)$  to be the union  $\bigcup_{\omega \in F} P_i(\omega)$ . We give an interpretation: if someone (“an analyst”) knows that some outcome in  $F$  happened (but does not know which one) then  $P_i(F)$  is the set of outcomes that  $i$  might consider possible (from the analyst’s perspective). Compute  $P_1(P_2(3))$  and  $P_2(P_1(3))$ .
- d. (10 points) We say an event  $E$  (a set of outcomes) is *common knowledge at  $\omega$*  if it is a superset of  $P_1(\omega)$ ,  $P_2(\omega)$ ,  $P_1(P_2(\omega))$ ,  $P_2(P_1(\omega))$ , and so on. Is the event  $E = \{1, 2, 3, 4, 5\}$  common knowledge at  $\omega = 3$ ?
- e. (10 points) Is the event  $E = \{8, 9, 10\}$  common knowledge at  $\omega = 9$ ?

**6. (25 points)** Explain where the reasoning of Osborne and Rubinstein's Section 5.1.3 would break down if some individual is not "perfectly rational"? Be precise, and say precisely where their chain of deductions breaks.

**7. (25 points)** Think back to the quiz where we tried to coordinate on common answers and found some "focal points." Explain why there wasn't common knowledge of focal points. If it wasn't common knowledge, what was it? Use the formalism you have learned to improve your explanation. Write about 250-300 words.

**8. (25 points)** Describe the setting of EK Ch. 16 – the information cascades model – in the language of Osborne and Rubinstein Ch. 5. What is the state space  $\Omega$ ? What do people know, in terms of partitions? Talk us through the updating of beliefs that people do based on their information, using the notation you develop.