Econ 1033	**Problem set 6**
Prof. Ben Golub	Due: **Wed., Sep 21, 2020 11:50pm**

This problem set is worth 100 points. You should attempt problems totaling at least 100 points, and submit solutions to all parts of those problems. For any subset $\mathcal{P}$ of problems totaling at least 100 points, your $\mathcal{P}$-score is defined to be the fraction of points you receive for problems in $\mathcal{P}$ (i.e., points earned on problems in $\mathcal{P}$ divided by possible point total for problems $\mathcal{P}$, which may be more than 100). We will make your score equal to your $\mathcal{P}$-score where $\mathcal{P}$ (the set of problems that count) is chosen optimally for you.

0. Read EK Chapter 6 to the end of 6.3. Also read EK 19 until you come to "Part (i): Clusters are Obstacles to Cascades." in Section 19.3

1. (10 points) EK Chapter 6 Exercise 2.

2. (10 points) EK Chapter 6 Exercise 4.

3. (10 points) EK Chapter 6 Exercise 5.

4. (30 points) Suppose a small firm is considering entering the market currently inhabited by an incumbent EK Chapter 17 monopolist. The actions of the small firm are Enter and Stay Out and the actions of the big firm are Acquiesce after Entry and Fight after Entry. Firms select their actions simultaneously.

- **a. (15 points)** Write a model in the spirit of Chapter 17 that says what happens for each combination of actions, and which makes sense given the words used above. The monopolist will probably use changes in price or something like it to affect the market.
- **b. (10 points)** Then use that model to fill in the payoffs of a two-by-two game.[1]
- **c. (5 points)** Describe what you expect to happen in this game and why.

5. (10 points) Easley and Kleinberg Chapter 19 Exercise 1.

6. (10 points) Easley and Kleinberg Chapter 19 Exercise 2.

7. (10 points) Easley and Kleinberg Chapter 19 Exercise 3.

8. (20 points) Easley and Kleinberg Chapter 19 Exercise 4.

9. (20 points) Easley and Kleinberg Chapter 19 Problem 5.

[1] You may look at Chapter 6 Appendix C for reference/inspiration, and especially Figure 6.27, though your payoffs needn't be (and probably shouldn't be) exactly theirs.

10. (30 points) We have studied one kind of network game. Here is another, quite different, game. We fix an undirected graph G on a finite set of players N. Each player in a *borrow-a-book network game* chooses whether to buy a book. His choice is denoted by $s_i \in \{0, 1\}$. A player's value of reading the book is 1. If the player does not buy the book, then he can freely borrow it from any of his neighbors who bought it.[2] If none of a player's neighbors has bought the book, then the player would prefer to pay the cost c of buying the book himself rather than not having access to the book. This is problem is a classic *free rider* problem, but defined on a network. Formally:

$$u_i(1, \boldsymbol{x}_{-i}) = 1 - c$$
$$u_i(0, \boldsymbol{x}_{-i}) = \begin{cases} 1 & \text{if } s_j = 1 \text{ for some } j \in N_i \\ 0 & \text{if } s_j = 0 \text{ for all } j \in N_i \end{cases}$$

where $1 > c > 0$ denotes the cost of buying the book. Here the first argument is player i's own action, and the second argument is a vector $\boldsymbol{x}_{-i}$ consisting of all actions of other players (it is just a list of the actions of all other players, excluding i). The set N_i denotes the set of i's neighbors in the graph G.

- **a. (10 points)** Describe all Nash equilibria[3] of the borrow-a-book game when G is a clique of 6 nodes (everyone is linked to everyone else).
- **b. (5 points)** Describe all Nash equilibria of a borrow-a-book game when G is a circle of 6 nodes (the graph is connected and everyone is linked to exactly two other players).
- **c. (15 points)** For a given graph G, let $m(G)$ be the minimum number of books bought in any Nash equilibrium of the borrow-a-book game played on graph G.[4] Suppose we obtain G' by adding some links to G (keeping the set of players fixed). Give an example where $m(G') > m(G)$ and explain why your example works. (Hint: it can be done with six nodes.)

[2] Indirect borrowing is not permitted, so the player cannot borrow a book from a neighbor of a neighbor.

[3] As usual, players' strategies are nonrandom: they simply choose action 0 or 1.

[4] That is, look at each equilibrium, and ask how many books are bought in that equilibrium. Then take the minimum of all those numbers.