Econ 1033	**Problem set 1**
Prof. Ben Golub	Due: **Wednesday, September 9, 2020 11:50pm**

This problem set is worth 100 points. You should choose 5 problems (write at the top of your problem set which ones you have chosen) and complete them.

1. (20 points)

a. (5 points) Easley and Kleinberg Chapter 2 Problem 1(a).

b. (7 points) Easley and Kleinberg Chapter 2 Problem 1(b).

c. (8 points) [short writing assignment, about 200 words] Give a clear description of an economic situation in which pivotality might be important: for example, you can sketch a situation in which it might matter to how much money people can make; or you can describe cases where a manager may care about it. Be creative, and also be precise in your description. "Economic" is meant to be fairly broad/permissive: as long as there are decisions, incentives, or transactions involved and you are precise about them and their relationship to pivotality, you will get credit.

2. (20 points)

a. (5 points) Easley and Kleinberg Chapter 2 Problem 2(a).

b. (7 points) Easley and Kleinberg Chapter 2 Problem 2(b).

c. (8 points) Give a clear description of an economic situation in which global gatekeeping might be important:[1]for example, you can sketch a situation in which it might matter to how much money people can make; or you can describe cases where a planner may care about it. Be creative, and also be precise in your description. "Economic" is meant to be fairly broad/permissive: as long as there are decisions, incentives, or transactions involved and you are precise about them and their relationship to gatekeeping, you will get credit.

3. (20 points) Let A be the adjacency matrix of an undirected graph G. The entries of this matrix are $A_{ij} = A_{ji} = 1$ if there is an edge $\{i, j\}$ in G, and 0 otherwise. Recall that a *walk* of length ℓ from i to j is a sequence of nodes $i_1, i_2, \ldots, i_{\ell+1}$ such that any consecutive pair of nodes is connected by an edge, where $i_1 = i$ (the starting node) and $i_{\ell+1} = j$, the ending node.[2] Show by induction that the number of walks of length ℓ from i to j is the (i, j) entry[3] of the matrix A^ℓ (i.e., A multiplied by itself A times).

4. (20 points) Easley and Kleinberg Chapter 2 Problem 3(b). Give a clear and rigorous construction and an airtight explanation of why it has the requisite property.

[1]Don't worry about the local kind unless you really want to.

[2]We say this walk has length ℓ because it involves taking ℓ steps, though there are $\ell + 1$ nodes involved. (Think through a little example.)

[3]That is, the entry in row i and column j.

5. (20 points) The professor gives Aniko and Barack a game to play. They can devise a strategy together. After they devise their strategy, they will be sent to separate rooms where they cannot communicate in any way. Their goal is to make as much money as possible as a team.

After they go to the rooms, the game works like this: The professor flips a standard, fair coin in each room. He lets each player know the outcome ("Heads" or "Tails") of his/her own coin. Each player then has to write down a guess about the outcome of the coin in the *other* room. If *both* of them guess the other player's coin toss correctly, they get \$1000 (split equally between them). Otherwise, they both get nothing. What is an optimal strategy for Aniko and Barack (which maximizes their joint expected earnings), and what are the expected total winnings they can make together? What is the strategy that *minimizes* their joint winnings?

6. (20 points) Read Easley and Kleinberg Section 21.2. For this exercise work in the model presented in that section. Define q_n to be the probability (evaluated before the epidemic starts) that the epidemic has not died out by wave n (i.e., that there are new people infected at wave n). Assume $0 < p < 1$ and $k \geq 1$.

- **a. (3 points)** Explain why $q_n < 1$ for all $n \geq 1$.
- **b. (5 points)** Explain why $q_n > q_{n+1}$ for all $n \geq 1$.
- **c. (6 points)** Explain three distinct ways in which the assumptions of the model in Section 21.2 are not a good fit to, say, the early Covid epidemic. This is a short writing assignment and you should aim to write about 200 words.
- **d. (6 points)** If $k = 2$ and $p = 0.1$, what is q_2, the probability that the epidemic has not died out by wave 2? Do the calculation manually, explaining your reasoning.