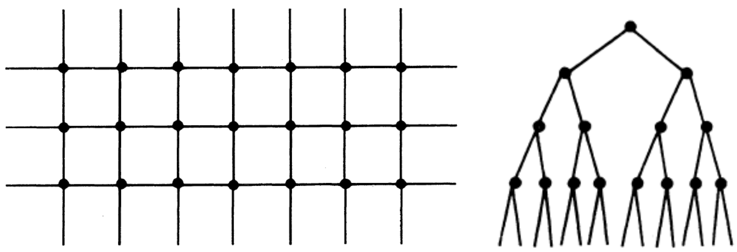
Econ 1033	**Problem set 7**
Prof. Ben Golub	Due: **Wednesday, October 28, 2020 at 11:50pm**

This problem set is worth 100 points. You should choose problems adding up to at least 100 points (write at the top of your problem set which ones you have chosen) and complete them.

Reading: [J-HN] Ch. 4, [EK] 19.6, Chwe's *Rational Ritual* pp. 1–33, 37–49, 61–66.

1. (30 points) Read Easley and Kleinberg Section 19.7 through the end of 19.7.A. Recall the definition of the *cascade capacity* from Chapter 19. In this exercise you will calculate the cascade capacity of some graphs. In each case, clearly explain why your answer is correct. This means both explaining why contagion occurs starting from a finite set at your claimed contagion threshold, and also why it does not occur for any higher value of q.



(a) (b)

Figure 1: Illustrations for Problem 4.

- **a. (5 points)** Show that in any infinite graph, the cascade capacity is at least $1/\max_{v \in N} d_v$, where d_v is the degree of node v and the maximum runs over all nodes v.
- **b. (5 points)** Let G be the infinite square lattice graph in 2D: The set of nodes V is the set of ordered pairs of integers (i, j). There is an edge between (i, j) and (i', j') if and only if (a) $|i' - i| = 1$ and $j' = j$ or (b) $|j' - j| = 1$ and $i' = i$. What is the cascade capacity?
- **c. (10 points)** Let G be the analogous infinite lattice graph in 3D: The set of nodes V is the set of ordered triples of integers (i, j, k). There is an edge between (i, j, k) and (i', j', k') if and only if exactly one of the following holds: (a) $|i' - i| = 1$ and $j' = j$ and $k' = k$; (b) $|j' - j| = 1$ and $i' = i$ and $k' = k$; or (c) $|k' - k| = 1$ and $i' = i$ and $j' = j$. What is the cascade capacity?
- **d. (10 points)** Let G be the infinite tree where each individual has one parent and 2 children. (Thus, every node except the root has degree 3.) What is the cascade capacity?

2. (30 points) Read [J-HN] Ch. 4 and the Chwe reading. Your response should be about 300 words for each part.

- **a. (15 points)** In terms of how you would model them, what are some salient differences between the way a biological contagion spreads and the way a financial contagion (of the type discussed in [J-HN] Ch. 4) spreads. Is the contagion model of [EK] Ch. 19 a useful starting point for thinking about financial contagions? How would you adapt that model to capture some of the considerations discussed in [J-HN] Ch. 4?

b. (15 points) Discuss how ideas from Chwe can be relevant in financial crises. (Think about whether people are willing to accept certain pieces of paper to fulfill debts, etc., but don't make this your only example.) Be specific. Look at some of the references Jackson cites if you need more specifics about the workings of financial crises.

3. (20 points) Write about 250 words explaining "finsta," etc., to Ben. Try to be a good anthropologist and convey some real insight on the phenomenon, but don't worry about making it sound academic or official.

4. (20 points) Do you buy what Chwe says about Super Bowl advertising and how it's related to common knowledge? Be a good social scientist and suggest at least one alternative explanation for why Superbowl ads may be more expensive *per eyeball*. Write about 300 words. Feel free to present a brief model.

5. (20 points) Summarize how Chwe relates common knowledge issues to strong links and weak links. Critique his argument, pointing out both what is convincing and where you think it may fall short of being fully convincing. Write about 300 words.

6. (30 points) Here you will develop in precise terms a generalization of the [EK] Ch. 19 basic contagion model to heterogeneous thresholds..

a. (10 points) Paralleling Section 19.2, explain why people might have different thresholds in terms of the underlying payoffs.

b. (10 points) Give a precise description of your contagion process with heterogeneous thresholds.

c. (10 points) Consider Claims (i) and (ii) in Section 19.3. Give a generalization of either one (your choice) that is valid for your generalized process.

7. (30 points) Do this problem if you're comfortable reading Easley and Kleinberg Section 19.7.B and wish to think through the subtleties of the cascade capacity on infinite graphs even before we cover it in class.

Let G be a *connected*, undirected graph specified by the pair (V, E), where V is an infinite set of nodes and E is a set of undirected edges (pairs of nodes). We will study a version of the Easley and Kleinberg Chapter 19 process, but we will no longer assume in this problem that the weights i places on all neighbors in G are equal. Let $\boldsymbol{W}$ be a matrix of nonnegative weights, with $w_{ij} = w_{ji}$ being the amount of time that i and j spend together. Assume that these objects, taken together, satisfy the following assumptions: (i) each node has only finitely many edges in G; (ii) $w_{ij} > 0$ if and only if i and j have an an edge between them; (iii) $w_{ij} \geq 0.01$ whenever it is nonzero.

In the contagion process we studied, the rule for updating is as follows. If the set of agents currently playing A at some time t is called $S(t)$, than an agent $i \notin S(t)$ switches at time $t+1$ from playing B to playing A if

$$\frac{\sum_{j \in S(t)} w_{ij}}{\sum_j w_{ij}} \geq q.$$

The denominator is simply the amount of *all* social time spent by i, while the numerator is the amount of social time spent with people playing A.

Define the cascade capacity as usual: the largest value of q so that best-response dynamics starting from a finite set of agents playing 1 (and all others playing 0) cause each individual to be infected eventually.[1]

- **a. (25 points)** Redo Easley and Kleinberg's proof for the claim in Section 19.7.B (the cascade capacity is at most 1/2) in the more general setting described above.
- **b. (5 points)** Discuss whether assumption (iii) was important in your proof, or whether you could have completed essentially the same proof without it.

[1] That is, they cause every agent i to play 1 for all times after some finite time t_i.