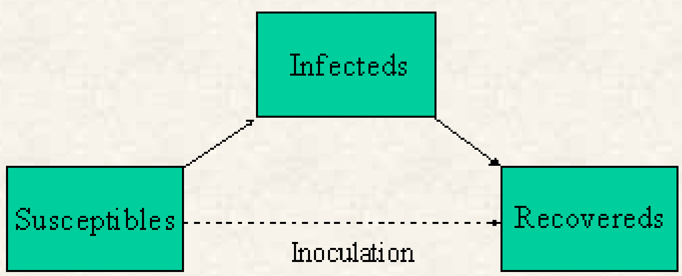
The SIR Model for Spread of Disease

Part 6: Herd Immunity

Each strain of flu is a disease that confers future immunity on its sufferers. For such a disease, if almost everyone has had it, then those who have *not* had it are protected from getting it -- there are not enough susceptibles left in the population to allow an epidemic to get under way. This group protection is called **herd immunity**.

In Part 4 you experimented with the relative sizes of **b** and **k**, and you found that, if **b** is small enough relative to **k**, then no epidemic can develop. In the language of Part 5, if the contact number **c = b/k** is small enough, then there will be no epidemic. But another way to prevent an epidemic is to reduce the initial susceptible population artificially by **inoculation**.

The point of inoculation is to create herd immunity by stimulating in as many people as possible the antibodies that confer immunity -- but without actually giving those people the disease. Thus inoculation creates a direct path from the susceptible group to the recovered group without passing through the infected group (see the diagram below). And a large-scale inoculation program to head off an impending epidemic does this rapidly enough to lower the initial susceptible population to a safe level -- safe enough that if a trace level of infection enters the population, a few people may get sick, but no epidemic will develop.



So, what fraction of the population must be inoculated to obtain herd immunity? Or, put another way, how small must $\mathbf{s_0}$ be to insure that an epidemic cannot get started? It depends on the contact number.

1. Explain why keeping an epidemic from getting started is the same as keeping **di/dt** negative from **t = 0** on.

2. Write the right-hand side of the infected-fraction differential equation

$$\frac{di}{dt} = b\,s(t)\,i(t) - k\,i(t)$$

in factored form. Explain why one factor is always positive and why the sign of other factor depends on the size of **s(t)**.

3. Explain why **s(t)** is a decreasing function, and thus has its largest value at **t = 0**. It follows that, if **di/dt** is negative at time **0**, then it *stays* negative.

4. Show that

$$i'(0) = (b\,s_0 - k)\,i_0.$$

Explain why, if $\mathbf{s_0}$ is less than **1/c**, then no epidemic can develop.

5. From 1912 to 1928, the contact number for measles in the U.S. was 12.8. If we assume that **c** is still 12.8 and that inoculation is 100% effective -- everyone inoculated obtains immunity from the disease -- what fraction of the population must be inoculated to prevent an epidemic?

6. Suppose the vaccine is only 95% effective. What fraction of the population would have to be inoculated to prevent a measles epidemic?